Day 74 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{4x^2} \times 2$.

$\sqrt{4x^2} = (4x^2)^{\frac{1}{2}} = 2x^1$

$2x^1 \times 2 = 4x$

Answer: 4x

2 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 8 m s^{-1} to 22 m s^{-1} as it travels 52.5 m. Find the acceleration.

$v^2 = u^2 + 2as$

$22^2 = 8^2 + 2 \times a \times 52.5$

$420 = 105a$

Answer: 4 m s^{-2}

3 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(5 + \sqrt{2})^2$.

$= 5^2 + 2 \times 5 \times \sqrt{2} + (\sqrt{2})^2$

$= 25 + 10\sqrt{2} + 2$

Answer: $27 + 10\sqrt{2}$

4 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 5 values: 20, 18, 15, 12, 4

Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 69$, $\Sigma x^2 = 1109$, $n = 5$

Mean $= 69 \div 5 = 13.80$

Variance $= \frac{1109}{5} - (13.80)^2 = 31.36$

Standard deviation $= \sqrt{31.36}$

Answer: mean = 13.8, standard deviation = 5.60

5 Estimate with a small angle (AQA, Edexcel, OCR, MEI, Eduqas)

Use $\sin\theta \approx \theta$ to estimate sin 2°, giving your answer to 3 significant figures.

Convert to radians: $2° = \frac{2\pi}{180} = 0.03491$ rad

$\sin 2° \approx 0.0349$

Answer: 0.0349

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O at constant speed 26 m s^{-1}.

At that instant car Q starts from rest at O with constant acceleration 4 m s^{-2} in the same direction. Find the time when Q catches up P.

Distances equal: $26t = \frac{1}{2} \times 4 \times t^2$

$t(26 - 2t) = 0$, so $t = 0$ or $t = 26 \div 2$

Answer: 13 s

7 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 2x)^{-2}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -2$, $y = 2x$

$= 1 + (-2)(2x) + \frac{(-2)((-3))}{2}(2x)^2$

$= 1 - 4x + 12x^2$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 - 4x + 12x^2$; $|x| < \frac{1}{2}$

8 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 3x^2 - 9x + 2$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 6x - 9$
$dy/dx = 0$ gives $x = -1$ or $x = 3$
Substitute: $y = 7$ at $x = -1$ and $y = -25$ at $x = 3$
Answer: A $(-1, 7)$, B $(3, -25)$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.6. If so, the probability that Sam is late is 0.8; otherwise it is 0.3.
Given that Sam is late, find the probability that it rains.

P(both) = 0.6 × 0.8 = 0.48; P(not first and second) = 0.4 × 0.3 = 0.12
P(second) = 0.48 + 0.12 = 0.6
P = 0.48 ÷ 0.6
Answer: $\frac{4}{5} = 0.8$

10 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \cos^2 x \, dx$.

$\cos 2\theta = 2\cos^2\theta - 1$, so $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Integrate term by term
Answer: $\frac{x}{2} + \frac{\sin 2x}{4} + c$