Day 73 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Scatter diagrams and correlation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The scatter diagram shows age of a car and value of the car for a group.
Describe the correlation.

The points show no pattern: as age of a car increases, value of the car shows no consistent trend.

Answer: no correlation

2 Exponential growth and decay model (All boards)

A car is worth £$V = 20000 \times 0.75^t$ after t years.
Find (a) its value after 5 years, (b) when it is worth £10000.
Give answers to 3 s.f.

(a) $V = 20000 \times 0.75^5 = 4750$

(b) $0.75^t = 0.5$, so $t \log 0.75 = \log 0.5$

$t = \dfrac{\log 0.5}{\log 0.75} = 2.41$

Answer: (a) £4750 (b) 2.41 years

3 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x + 3)^2 + (y - 4)^2 = 49$.
Determine whether the point P(8, 2) is inside, outside or on the circle.

Distance^2 from the centre $(-3, 4)$ to P:

$(8 - (-3))^2 + (2 - 4)^2 = 121 + 4 = 125$

$r^2 = 49$, and $125 > 49$.

Answer: P is outside the circle

4 Laws of logarithms: single log (All boards)

Given that $\log_3 x = p$, write the following in terms of p:

(a) $\log_3 x^3$ (b) $\log_3 (9x)$

(a) $\log_3 x^3 = 3 \log_3 x = 3p$

(b) $\log_3 (9x) = \log_3 9 + \log_3 x = 2 + p$

Answer: (a) 3p (b) p + 2

5 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 16 m s^{-1}.

At that instant car B starts from rest at O with constant acceleration 4 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $16t = \frac{1}{2} \times 4 \times t^2$

$t(16 - 2t) = 0$, so $t = 0$ or $t = 16 \div 2$

Answer: 8 s

6 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, where f(x) is a cubic. Find f(x) in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -3$, 3 and 4.

$f(x) = k(x + 3)(x - 3)(x - 4)$

At $(0, -36)$: $k \times 3 \times (-3) \times (-4) = -36$

$k = -1$

Answer: $f(x) = -(x + 3)(x - 3)(x - 4)$

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7 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. $\cos\alpha = \frac{4}{5}$. Find P and Q.

Horizontally: $P = 10\cos\alpha = 10 \times \frac{4}{5}$

Vertically: $Q = 10\sin\alpha = 10 \times \frac{3}{5}$

Answer: P = 8, Q = 6

8 Partial fractions: find the constants (All boards)

Express $\frac{x-2}{(x+1)(x+2)}$ in the form $\frac{A}{x+1} + \frac{B}{x+2}$.

Find the values of A and B.

$x - 2 \equiv A(x+2) + B(x+1)$

$x = -1$: $-3 = A(1)$, so $A = -3$

$x = -2$: $-4 = B(-1)$, so $B = 4$

Answer: A = −3, B = 4

9 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$(x + 3)$ is a factor of $x^3 + kx^2 - 3x - 27$.

Find the value of k.

Let $f(x) = x^3 + kx^2 - 3x - 27$. Then $f(-3) = 0$:

$-27 + 9k - 18 = 0$

$9k = 45$

Answer: k = 5

10 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

20 pairs of data on height and arm span have $r = -0.29$.

Test, at the 5% significance level, for evidence of negative correlation. The critical value is -0.3783.

$H_0: \rho = 0$, $H_1: \rho < 0$

$r = -0.29$ is not beyond the critical value -0.3783

Do not reject H_0

Answer: Do not reject H_0: there is not enough evidence of negative correlation