Day 72 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 6) and (2, $\frac{3}{2}$).

Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 6$

$x = 2$: $6 \times b^2 = \frac{3}{2}$, so $b^2 = \frac{1}{4}$

$b = \frac{1}{2}$

Answer: $a = 6$, $b = \frac{1}{2}$

2 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $2x^3 + x^2 - 3x + 2$ is divided by $(2x - 1)$.

Put $2x - 1 = 0$: $x = \frac{1}{2}$

Remainder = $f(\frac{1}{2}) = \frac{1}{4} + \frac{1}{4} + (-\frac{3}{2}) + 2$

$= 1$

Answer: 1

3 Stationary points of a cubic (All boards)

$y = x^3 - 3x^2 + 2$.

Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 6x = 0$

$x = 0$: $y = 2$; $x = 2$: $y = -2$

$d^2y/dx^2 = 6x - 6$: at $x = 0$ it is -6 (maximum), at $x = 2$ it is 6 (minimum)

Answer: (0, 2) maximum; (2, −2) minimum

4 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 7x + 10 \geq 0$.

Solve $x^2 - 7x + 10 = 0$: $(x - 2)(x - 5) = 0$

Critical values $x = 2$ and $x = 5$.

The graph is a ∪-shaped parabola, so take the regions outside the roots.

Answer: $x \leq 2$ or $x \geq 5$

5 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(7\mathbf{j})$ N and $(4\mathbf{i} - 5\mathbf{j})$ N act on a particle of mass 2 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(7\mathbf{j}) + (4\mathbf{i} - 5\mathbf{j})$

$= (4\mathbf{i} + 2\mathbf{j})$ N

$F = ma$: $a = \frac{1}{2}(4\mathbf{i} + 2\mathbf{j})$

Answer: $(2\mathbf{i} + \mathbf{j})$ m s^{-2}

6 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.5. If so, the probability that an item is faulty is 0.4; otherwise it is 0.1. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = $0.5 \times 0.4 = 0.2$; P(not first and second) = $0.5 \times 0.1 = 0.05$

P(second) = $0.2 + 0.05 = 0.25$

$P = 0.2 \div 0.25$

Answer: $\frac{4}{5} = 0.8$

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7 Lift: tension in the cable (All boards)

A lift of mass 500 kg is raised by a vertical cable with constant acceleration 0.75 m s^{-2} upwards. Take g = 9.8 m s^{-2}. Find the tension in the cable.

Resolving upwards: $T - 500 \times 9.8 = 500 \times 0.75$

$T = 4900 + 375$

Answer: 5275 N

8 Normal approximation to the binomial (CIE)

X ~ B(100, 0.2). Use a normal approximation to find $P(X \leq 24)$, to 3 s.f.

np = 20 and npq = 16, so use Y ~ N(20, 16)

Continuity correction: P(Y < 24.5)

$z = (24.5 - 20) \div \sqrt{16}$

Answer: 0.870

9 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which

$x^2 + kx + 49 = 0$ has no real roots.

$b^2 - 4ac < 0$

$k^2 - 196 < 0$, so $k^2 < 196$

Answer: $-14 < k < 14$

10 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 3\mathbf{i} - 3\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (3)(-3) + (0)(2) + (-3)(-1) = -6$

$|\mathbf{a}| = \sqrt{18}$, $|\mathbf{b}| = \sqrt{14}$

$\cos\theta = \frac{-6}{\sqrt{18} \times \sqrt{14}}$, so $\theta = 112.2°$

Answer: 112.2°