Day 70 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Velocity-time: constant deceleration to rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car decelerates uniformly from 10 m s^{-1} to rest in 4 s. Find (a) the deceleration, (b) the distance travelled while decelerating.

(a) $a = \frac{v - u}{t} = \frac{0 - 10}{4}$

(b) $s = \frac{(u + v)}{2} t = 5 \times 4$

Answer: (a) 2.5 m s^{-2} (b) 20 m

2 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x - 3)^2 - 6$, $x \geq 0$.
Find the range of f.

The minimum of $(x - 3)^2$ is 0 at $x = 3$, which is allowed since $3 \geq 0$.
So $f(x) \geq -6$.
Answer: $f(x) \geq -6$

3 Solve with a shifted angle (All boards)

Solve $\sin(x - 20°) = -\frac{\sqrt{2}}{2}$ for $0° \leq x \leq 360°$.

Let $u = x - 20°$, so $-20° \leq u \leq 340°$

$\sin u = -\frac{\sqrt{2}}{2}$: $u = 225°, 315°$

$x = u + 20°$: $x = 245°, 335°$
Answer: x = 245°, 335°

4 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cap B) = 0.15$.
(a) Find $P(A \mid B)$.
(b) Are A and B independent? Give a reason.

(a) $P(A \mid B) = P(A \cap B) \div P(B) = 0.15 \div 0.5 = 0.3$
(b) $P(A) \times P(B) = 0.3 \times 0.5 = 0.15$
This equals $P(A \cap B) = 0.15$, so A and B are independent
Answer: (a) 0.3 (b) independent, as P(A) × P(B) = P(A ∩ B)

5 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 7$.
Use the Newton-Raphson method with $x_0 = 3$ to find x_1,
giving your answer to 3 decimal places.

$f'(x) = 2x$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 3 - \frac{2.000}{6.000} = 2.6667$

Answer: $x_1 = 2.667$

6 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the range of values of k for which $kx^2 + 6x + 3 = 0$ has real roots.

Real roots: $b^2 - 4ac \geq 0$
$36 - 12k \geq 0$, so $k \leq 3$
Also $k \neq 0$ for the equation to be a quadratic.
Answer: $k \leq 3$, $k \neq 0$

7 Car towing a caravan (All boards)

A car of mass 1000 kg tows a caravan of mass 500 kg on a level road with acceleration 0.5 m s^{-2}. Resistances: 400 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 400 - 100 = 1500 \times 0.5$

Caravan: $T - 100 = 500 \times 0.5$

Answer: D = 1250 N, T = 350 N

8 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.

Find (a) the equation of AB in the form px + qy = r with integers p, q and r, (b) the length AB.

Gradient = $\frac{-4-0}{0-(-1)} = -4$

Using y = mx + c with c = −4: y = −4x − 4, so 4x + y = −4

$AB^2 = 1^2 + 4^2 = 17$

Answer: (a) 4x + y = −4 (b) $\sqrt{17}$

9 Identity then solve (sin² to cos) (All boards)

Solve $-2\sin^2 x - \cos x + 2 = 0$

for $0° \leq x \leq 360°$.

Use $\sin^2 x = 1 - \cos^2 x$: $2\cos^2 x - \cos x = 0$

Factorise and solve: $\cos x = \frac{1}{2}$ or $\cos x = 0$

x = 60°, 90°, 270°, 300°

Answer: x = 60°, 90°, 270°, 300°

10 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 10 to 30 has frequency 60 and is drawn with a bar of height 3 cm.

The class 30 to 35 has frequency 25. Find the height of its bar.

Frequency density = frequency ÷ class width

First bar: 60 ÷ 20 = 3, so 3 cm represents 3

Second bar: 25 ÷ 5 = 5

Height = $3 \times \frac{5}{3}$

Answer: 5 cm