Day 7 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -3\mathbf{i} + 4\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{3^2 + 4^2} = 5$

(b) Acute angle = $\tan^{-1}(\frac{4}{3}) = 53.1°$

p is in the second quadrant, so the angle is 126.9°

Answer: (a) 5 (b) 126.9°

2 Addition rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.45, P(B) = 0.7 and P(A ∪ B) = 0.75.
(a) Find P(A ∩ B).
(b) Find P(A′ ∩ B′).

(a) P(A ∩ B) = P(A) + P(B) − P(A ∪ B) = 0.45 + 0.7 − 0.75 = 0.4
(b) A′ ∩ B′ is outside both, so P = 1 − P(A ∪ B) = 1 − 0.75

Answer: (a) 0.4 (b) 0.25

3 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 27 terms of the series
3 + 7 + 11 + 15 + ...

a = 3, d = 4, n = 27

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{27}{2}[2(3) + 26 \times 4]$

= 1485

Answer: 1485

4 Addition formulae: exact values (All boards)

By writing 15° as 60° − 45°, find the exact value of cos 15°.

cos(60° − 45°) = cos 60° cos 45° + sin 60° sin 45°

$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

Simplify: $\frac{\sqrt{6} + \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} + \sqrt{2}}{4}$

5 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 0 and 20 minutes.

Frequency = frequency density × class width
0 ≤ t < 10: 1 × 10 = 10
10 ≤ t < 20: 6 × 10 = 60
Total = 10 + 60

Answer: 70

6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−4, 1) and B is (−8, 3).
Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (−6, 2)

Gradient of AB = $\frac{2}{-4} = -\frac{1}{2}$, so perpendicular gradient = 2

y − 2 = 2(x − (−6))
Multiply out and rearrange.

Answer: 2x − y = −14

7 Power of an engine: driving force (CIE)

A car's engine works at a constant rate of 45 kW. Find the driving force when the vehicle is moving at 15 m s^{-1}.

$P = Fv$

$45000 = F \times 15$

Answer: 3000 N

8 Rewrite a sin x + b cos x in R form (All boards)

Express 15 sin x + 8 cos x in the form R cos(x − α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

$R\cos(x - \alpha) = R\cos x \cos\alpha + R\sin x \sin\alpha$

$R\cos\alpha = 8$ and $R\sin\alpha = 15$

$R = \sqrt{15^2 + 8^2} = 17$, $\tan\alpha = \frac{15}{8}$, $\alpha = 61.9°$

Answer: 17 cos(x − 61.9°)

9 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^3 - 6x^2 + 13x - 25$ by (x − 4).
Find the quotient and the remainder.

Use synthetic division with a = 4:

Bottom row: 1, −2, 5, −5

Quotient: $x^2 - 2x + 5$

Remainder: −5

Answer: Quotient $x^2 - 2x + 5$, remainder −5

10 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 24 m s^{-1} brakes uniformly and comes to rest after travelling 60 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 24^2 + 2 \times a \times 60$

so $a = -4.8$, a deceleration of 4.8

(b) $s = \frac{(u + v)}{2} t$: $60 = 12\,t$

Answer: (a) 4.8 m s^{-2} (b) 5 s