Day 69 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x = 4t^2 + 5$, $y = 2t^3 - t$.
Find dy/dx in terms of t, and its value when t = 1.

$dx/dt = 8t$, $dy/dt = 6t^2 - 1$

$dy/dx = \frac{dy}{dt \div dx/dt} = \frac{6t^2 - 1}{8t}$

At t = 1: $dy/dx = \frac{5}{8}$

Answer: $dy/dx = \frac{6t^2 - 1}{8t}$; $\frac{5}{8}$

2 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 4)^2 + (y + 2)^2 = 16$.
Determine whether the point P(−7, 3) is inside, outside or on the circle.

Distance^2 from the centre (4, −2) to P:
$(-7 - 4)^2 + (3 - (-2))^2 = 121 + 25 = 146$
$r^2 = 16$, and $146 > 16$.
Answer: P is outside the circle

3 Parallel vectors (All boards)

$\mathbf{a} = \begin{pmatrix}5\\3\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}20\\12\end{pmatrix}$.
Show that a and b are parallel and find the ratio |b| : |a|.

b = 4 × a, since 4 × 5 = 20 and 4 × 3 = 12
So a and b are parallel and |b| : |a| = 4 : 1
Answer: b = 4 a, so parallel; ratio 4 : 1

4 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle starts from rest and moves in a straight line with constant acceleration 4 m s^{-2}.
It travels 32 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with u = 0

$32 = \frac{1}{2} \times 4 \times t^2$

$t^2 = 16$
Answer: 4 s

5 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by considering the possible remainders, that the square of any integer is of the form 4k or 4k + 1.

Any integer n has the form 4k + r where r = 0, 1, ..., 3.
$(4k)^2 = 16k^2 = 4(4k^2)$
$(4k + 1)^2 = 16k^2 + 8k + 1 = 4(4k^2 + 2k) + 1$
$(4k + 2)^2 = 16k^2 + 16k + 4 = 4(4k^2 + 4k + 1)$
$(4k + 3)^2 = 16k^2 + 24k + 9 = 4(4k^2 + 6k + 2) + 1$
Every case is of the required form.
Answer: Shown

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6 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 8t + 12)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 8t + 12 = 0$
$(t - 2)(t - 6) = 0$
(b) $a = \frac{dv}{dt} = 2t - 8$; at $t = 2$, $a = 4 - 8$

Answer: (a) $t = 2$ and $t = 6$ (b) -4 m s^{-2}

7 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(15, p)$. H_0: $p = 0.25$, H_1: $p > 0.25$.
Find the critical region for a test at the 10% significance level.

$P(X \geq 7) = 0.0566 \leq 0.1$
$P(X \geq 6) = 0.148 > 0.1$
Answer: $X \geq 7$

8 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(5 - \sqrt{7})(7 + \sqrt{7})$.

$= 35 + 5\sqrt{7} - 7\sqrt{7} - 7$
$= 28 - 2\sqrt{7}$
Answer: $28 - 2\sqrt{7}$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.2. If so, the probability that Sam is late is 0.3; otherwise it is 0.1.
Given that Sam is late, find the probability that it rains.

P(both) = 0.2 × 0.3 = 0.06; P(not first and second) = 0.8 × 0.1 = 0.08
P(second) = 0.06 + 0.08 = 0.14
P = 0.06 ÷ 0.14
Answer: $\frac{3}{7}$ = 0.429

10 Stationary points of a cubic (All boards)

$y = x^3 - 3x^2 - 9x + 5$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 6x - 9 = 3(x + 1)(x - 3) = 0$
$x = -1$: $y = 10$; $x = 3$: $y = -22$
$d^2y/dx^2 = 6x - 6$: at $x = -1$ it is -12 (maximum), at $x = 3$ it is 12 (minimum)
Answer: $(-1, 10)$ maximum; $(3, -22)$ minimum