Day 68 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Scatter diagrams and correlation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The scatter diagram shows distance from city and house price for a group.
Describe the correlation.

The points lie close to a line with negative gradient.
As distance from city increases, house price tends to decrease.
Answer: strong negative correlation

2 Simplify using identities (All boards)

Simplify $\cos^2 x - 1$.

$\cos^2 x - 1 = -(1 - \cos^2 x) = -\sin^2 x$
Answer: $-\sin^2 x$

3 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2(x + 2)(x - 1)(x - 5)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -2, 1, 5$
$x = 0$: $y = 2 \times (-1) \times (-5) \times 2 = 20$
Answer: $(-2, 0)$, $(1, 0)$, $(5, 0)$, $(0, 20)$

4 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{1}{2}x$ for $0 \leq x \leq 2$, and 0 otherwise.

Find E(X).

$E(X) = \int$ from 0 to 2 of $x\, f(x)\, dx = \int$ from 0 to 2 of $\frac{1}{2}x^2\, dx$

$= \frac{1}{2} \times \frac{2^3}{3}$

Answer: $\frac{4}{3}$

5 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 3x + 4$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 3$
$dy/dx = 0$ gives $x = -1$ or $x = 1$
Substitute: $y = 6$ at $x = -1$ and $y = 2$ at $x = 1$
Answer: A $(-1, 6)$, B $(1, 2)$

6 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 3x)^{-3}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n - 1)}{2!}y^2 + \ldots$ with $n = -3$, $y = 3x$

$= 1 + (-3)(3x) + \frac{(-3)((-4))}{2}(3x)^2$

$= 1 - 9x + 54x^2$

Valid when $|3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 - 9x + 54x^2$; $|x| < \frac{1}{3}$

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7 Constant acceleration: finding the initial speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration 0.5 m s^{-2} travels 40 m in 4 s.
Find its initial speed.

$s = ut + \frac{1}{2}at^2$

$40 = 4u + \frac{1}{2} \times 0.5 \times 4^2$

$4u = 36$

Answer: 9 m s^{-1}

8 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 8 terms of the geometric series 5 + 10 + 20 + ...

$a = 5, r = 2, n = 8$

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{5(2^8 - 1)}{2 - 1}$

$= 1275$

Answer: 1275

9 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 5 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}.
Find (a) its acceleration, (b) its speed after 4 s.

(a) Parallel to slope: $mg\sin\alpha = ma$

$a = 9.8 \times \frac{3}{5}$

(b) $v = u + at = 0 + 5.88 \times 4$

Answer: (a) 5.88 m s^{-2} (b) 23.52 m s^{-1}

10 Product rule (All boards)

Use the product rule to differentiate
$y = x^2e^{2x}$.

$u = x^2, v = e^{2x}$

$u' = 2x^1, v' = 2e^{2x}$

$dy/dx = u'v + uv'$, which simplifies to the answer below

Answer: $dy/dx = x^1e^{2x}(2x + 2)$