Day 65 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Natural logs and e^x (All boards)

Solve $e^{3x+3} = 12$.
Give your answer to 3 significant figures.

Take natural logs: $3x + 3 = \ln 12 = 2.485$
$x = -0.172$
Answer: $x = -0.172$

2 Tension in a string supporting a hanging mass (All boards)

A particle of mass 3 kg hangs from a light string and is raised at constant speed. Taking $g = 9.8$ m s^{-2}, find the tension in the string.

No acceleration, so $T = mg$
$T = 3 \times 9.8$
Answer: 29.4 N

3 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the first four terms, in ascending powers of x, of the expansion of $(1 + x)^4$.

$(1 + y)^4 = 1 + 4y + 6y^2 + 4y^3 + \ldots$ with $y = x$
$= 1 + 4(x) + 6(x)^2 + 4(x)^3$
Answer: $1 + 4x + 6x^2 + 4x^3$

4 Addition rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, $P(A) = 0.45$, $P(B) = 0.7$ and $P(A \cup B) = 0.8$.
(a) Find $P(A \cap B)$.
(b) Find $P(A' \cap B')$.

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.45 + 0.7 - 0.8 = 0.35$
(b) $A' \cap B'$ is outside both, so $P = 1 - P(A \cup B) = 1 - 0.8$
Answer: (a) 0.35 (b) 0.2

5 Friction: acceleration along rough ground (All boards)

A block of mass 2 kg is pulled along rough horizontal ground by a horizontal force of 8.92 N.
The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the acceleration.

$R = 2 \times 9.8 = 19.6$
$F = \mu R = 0.2 \times 19.6 = 3.92$
$P - F = ma$: $8.92 - 3.92 = 2\,a$
Answer: 2.5 m s^{-2}

6 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with temperature (°C) from 10 to 22, the regression line is $S = 6 + 8T$.
Estimate ice cream sales when $T = 16$ and comment on the reliability.

$S = 6 + 8 \times 16 = 134$
16 is inside the range 10 to 22, so this is interpolation.
Answer: S = 134; estimate is reliable, as 16 is within the data range (interpolation)

7 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 5 and common difference 4. Find the smallest n for which the sum of the first n terms exceeds 800.

$S_n = \frac{n}{2}[10 + 4(n - 1)] = \frac{n(4n + 6)}{2}$
$S_{19} = 779$ and $S_{20} = 860$
The sum first exceeds 800 when $n = 20$.
Answer: n = 20

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8 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y^2 = 3x - 2$.
Find dy/dx in terms of x and y, and hence the gradient at (2, 2).

Differentiate both sides with respect to x (using dy/dx for y terms)

dy/dx = $\frac{3}{2y}$

At (2, 2): dy/dx = $\frac{3}{4}$

Answer: dy/dx = $\frac{3}{2y}$; gradient $\frac{3}{4}$

9 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - 9x + 10$, $y = -2x - 2$.

$x^2 - 9x + 10 = -2x - 2$
$x^2 - 7x + 12 = 0$
$(x - 4)(x - 3) = 0$
Use $y = -2x - 2$ to find each y.
Answer: x = 3, y = −8 and x = 4, y = −10

10 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = -2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{j} + \mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

a.b = (−2)(0) + (2)(−2) + (3)(1) = −1
|a| = $\sqrt{17}$, |b| = $\sqrt{5}$
cos θ = $\frac{-1}{\sqrt{17} \times \sqrt{5}}$, so θ = 96.2°
Answer: 96.2°