Day 64 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 3 ln x
(b) $3e^x + 3\ln x$

(a) $dy/dx = \frac{3}{x}$

(b) $dy/dx = 3e^x + \frac{3}{x}$

Answer: (a) $\frac{3}{x}$ (b) $3e^x + \frac{3}{x}$

2 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−5, −6) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = 4f(x) (b) y = f(x + 5)

(a) y = 4f(x) multiplies y by 4, so P maps to (−5, −24).
(b) y = f(x + 5) subtracts 5 from x, so P maps to (−10, −6).
Answer: (a) (−5, −24) (b) (−10, −6)

3 Normal distribution: probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The scores, X, in a test are modelled by $N(95, 5^2)$.
Find P(97.5 < X < 105) to 3 s.f.

z-values: 0.5 and 2
P(0.5 < Z < 2) from calculator
Answer: 0.286

4 Lift: tension in the cable (All boards)

A lift of mass 500 kg is lowered by a vertical cable with constant acceleration 0.75 m s^{-2} downwards. Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolving downwards: 500 × 9.8 − T = 500 × 0.75
T = 4900 − 375
Answer: 4525 N

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

12 pairs of data on daily mean temperature and sunshine hours have r = 0.47.
Test, at the 5% significance level, for evidence of correlation. The critical value is ±0.5760.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
r = 0.47 is not beyond the critical value ±0.5760
Do not reject H_0
Answer: Do not reject H_0: there is not enough evidence of correlation

6 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - 15$, $y = -x - 3$.

$x^2 - 15 = -x - 3$
$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
Use $y = -x - 3$ to find each y.
Answer: x = −4, y = 1 and x = 3, y = −6

7 Separable differential equations (All boards)

Given that $dy/dx = \frac{x}{y}$ and $y = 1$ when $x = 0$,

find y^2 in terms of x.

$y\,dy = x\,dx$, so $\frac{y^2}{2} = \frac{x^2}{2} + c$

$x = 0, y = 1$: $c = \frac{1}{2}$

$y^2 = x^2 + 1$

Answer: $y^2 = x^2 + 1$

8 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both

$3x - 1 < -16$ and $x^2 + 7x + 6 < 0$.

$3x - 1 < -16$ gives $x < -5$.

$x^2 + 7x + 6 < 0$: $(x + 6)(x + 1) < 0$, so $-6 < x < -1$.

Both are true where the two ranges overlap.

Answer: $-6 < x < -5$

9 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity

$(-2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. It is at the point with position vector $(-4\mathbf{i} + 2\mathbf{j})$ m at $t = 0$. Find the time when its position vector is $(-18\mathbf{i} + 23\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

$(-18\mathbf{i} + 23\mathbf{j}) = (-4\mathbf{i} + 2\mathbf{j}) + t(-2\mathbf{i} + 3\mathbf{j})$

Equating i components: $-18 = -4 + -2t$

Answer: 7 s

10 Degrees and radians (All boards)

(a) Convert 210° to radians, giving your answer in terms of π.

(b) Convert $\frac{\pi}{12}$ radians to degrees.

(a) $210° = 210 \times \frac{\pi}{180} = \frac{7\pi}{6}$

(b) $\frac{\pi}{12}$ radians $= \frac{\pi}{12} \times \frac{180}{\pi} = 15°$

Answer: (a) $\frac{7\pi}{6}$ radians (b) 15°