Day 63 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Types of data (AQA, Edexcel, OCR, MEI, Eduqas)

Classify each variable as qualitative, discrete or continuous.
(a) the number of cars in a car park
(b) the make of a car
(c) the eye colour of a pupil

(a) discrete: it is counted, so takes separate values
(b) qualitative: it is described by words, not numbers
(c) qualitative: it is described by words, not numbers
Answer: (a) discrete; (b) qualitative; (c) qualitative

2 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 12 m s^{-1} brakes uniformly and comes to rest after travelling 36 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 12^2 + 2 \times a \times 36$
so $a = -2$, a deceleration of 2

(b) $s = \frac{(u + v)}{2}$ t: $36 = 6$ t

Answer: (a) 2 m s^{-2} (b) 6 s

3 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = x(3 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where $x = 0$ and $x = 3$

Area $= \int_0^3 3x - x^2 \, dx = [\frac{3x^2}{2} - \frac{x^3}{3}]$ from 0 to 3

$= \frac{3^3}{2} - \frac{3^3}{3}$

$= \frac{9}{2}$

Answer: $\frac{9}{2}$

4 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 5 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin \alpha = \frac{4}{5}$. Take $g = 9.8$ m s^{-2}.
Find (a) its acceleration, (b) its speed after 3 s.

(a) Parallel to slope: $mg \sin \alpha = ma$

$a = 9.8 \times \frac{4}{5}$

(b) $v = u + at = 0 + 7.84 \times 3$

Answer: (a) 7.84 m s^{-2} (b) 23.52 m s^{-1}

5 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form $px + qy = r$ with integers p, q and r, (b) the length AB.

Gradient $= \frac{-6 - 0}{0 - (-5)} = -\frac{6}{5}$

Using $y = mx + c$ with $c = -6$: $y = -\frac{6}{5}x - 6$, so $6x + 5y = -30$

$AB^2 = 5^2 + 6^2 = 61$

Answer: (a) $6x + 5y = -30$ (b) $\sqrt{61}$

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6 Rewrite a sin x + b cos x in R form (All boards)

Express 4 sin x + 3 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

R sin(x + α) = R sin x cos α + R cos x sin α
R cos α = 4 and R sin α = 3
$R = \sqrt{4^2 + 3^2} = 5$, $\tan \alpha = \frac{3}{4}$, α = 36.9°

Answer: 5 sin(x + 36.9°)

7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are x = 1 and y = 2, so a = 1 and c = 2.
Through (0, 4): $4 = \frac{k}{0 - 1} + 2$
$\frac{k}{-1} = 2$, so k = −2

Answer: a = 1, c = 2, k = −2

8 Exponential growth and decay model (All boards)

A radioactive sample has mass $M = 100\, e^{-0.05t}$ grams after t days.
Find the time for the mass to halve, to 3 significant figures.

$\frac{1}{2} = e^{-0.05t}$
$-0.05t = \ln \frac{1}{2} = -\ln 2$
$t = \frac{\ln 2}{0.05} = 13.9$ days

Answer: t = 13.9 days

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = −3x + k is a tangent to the curve
$y = x^2 - x + 5$.
Find the value of k.

$x^2 - x + 5 = -3x + k$
$x^2 + 2x + (5 - k) = 0$
A tangent touches once: discriminant = 0
$(2)^2 - 4(5 - k) = 0$

Answer: k = 4

10 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{1}{2}x$ for 0 ≤ x ≤ 2, and 0 otherwise.

Find P(X < 1).

$P(X < 1) = \int$ from 0 to 1 of $\frac{1}{2}x\, dx$

$= [\frac{1}{4}x^2]$ from 0 to 1

Answer: $\frac{1}{4}$