Day 62 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Definite integrals (exact) (All boards)

Evaluate

$\int_0^3 (x^3 + 2)\,dx$.

$[\frac{x^4}{4} + 2x]$ from 0 to 3

$= \frac{81}{4} + 6 = \frac{105}{4}$

Answer: $\frac{105}{4}$

2 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A) = 0.9, P(B) = 0.6 and P(A ∩ B) = 0.15.
(a) Find P(A | B).
(b) Are A and B independent? Give a reason.

(a) P(A | B) = P(A ∩ B) ÷ P(B) = 0.15 ÷ 0.6 = 0.25
(b) P(A) × P(B) = 0.9 × 0.6 = 0.54
This does not equal P(A ∩ B) = 0.15, so A and B are not independent
Answer: (a) 0.25 (b) not independent, as P(A) × P(B) ≠ P(A ∩ B)

3 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a car over 26 seconds as it moves in a straight line.
(a) Find its acceleration in the first 10 s.
(b) Find the total distance travelled.

(a) acceleration = gradient = $\frac{15}{10}$

(b) distance = area of trapezium

$= \frac{1}{2}(12 + 26) \times 15$

Answer: (a) 1.5 m s^{-2} (b) 285 m

4 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 2x + 4 and g(x) = 4x − 1.
Solve fg(x) = 2.

fg(x) = f(4x − 1) = 8x + 2
8x + 2 = 2
Answer: x = 0

5 Normal distribution: diagram and symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the density of X ~ N(35, 2^2).
a and b are equal distances from the mean. The shaded area is 0.42.
Find P(X > b).

Unshaded area = 1 − 0.42 = 0.58
The two tails are equal by symmetry, so P(X > b) = 0.58 ÷ 2
Answer: 0.29

6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (6, 4) and B is (2, 10).
Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (4, 7)

Gradient of AB = $\frac{6}{-4} = -\frac{3}{2}$, so perpendicular gradient = $\frac{2}{3}$

$y - 7 = \frac{2}{3}(x - 4)$

Multiply out and rearrange.
Answer: 2x − 3y = −13

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7 Angle between vectors (scalar product) (CIE)

a = $-2\mathbf{i}$ and b = $4\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

a.b = $(-2)(4) + (0)(4) + (0)(4) = -8$
$|a| = \sqrt{4}$, $|b| = \sqrt{48}$
$\cos\theta = \frac{-8}{\sqrt{4} \times \sqrt{48}}$, so $\theta = 125.3°$
Answer: 125.3°

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that if n^2 is even, then n is even.
Begin: "Assume n is odd, so n = 2k + 3 for an integer k."

$n^2 = (2k + 3)^2 = 4k^2 + 12k + 9$
$= 2(2k^2 + 6k + 4) + 1$
This is odd, which contradicts n^2 being even.
So the assumption is false: n must be even.
Answer: Proved

9 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity $(-3\mathbf{i} + 4\mathbf{j})$ m s^{-1} and constant acceleration $(-2\mathbf{i} - 3\mathbf{j})$ m s^{-2}.
Find its velocity after 5 s.

v = u + at
$\mathbf{v} = (-3\mathbf{i} + 4\mathbf{j}) + 5(-2\mathbf{i} - 3\mathbf{j})$
$= (-3\mathbf{i} + 4\mathbf{j}) + (-10\mathbf{i} - 15\mathbf{j})$
Answer: $(-13\mathbf{i} - 11\mathbf{j})$ m s^{-1}

10 Solve a logarithmic equation (two logs) (All boards)

Solve log x + log (x − 1) = log 20, x > 1.

log [x(x − 1)] = log 20, so $x^2 - 1x = 20$
$x^2 - 1x - 20 = 0$, $(x - 5)(x + 4) = 0$
x = 5 or x = −4; x > 1, so x = 5
Answer: x = 5