Day 60 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) e^{4x} + sin 5x

(b) 4 ln x

(a) dy/dx = 4 e^{4x} + 5 cos 5x

(b) dy/dx = $\frac{4}{x}$

Answer: (a) 4 e^{4x} + 5 cos 5x (b) $\frac{4}{x}$

2 Sampling: advantages (AQA, Edexcel, OCR, MEI, Eduqas)

A researcher wants to find out the health of the residents of a village.

Give one advantage and one disadvantage of using systematic sampling.

Advantage: it is quick and easy and spreads the sample evenly through the list.

Disadvantage: a repeating pattern in the list could cause bias, and a list is needed.

Answer: Advantage: quick and evenly spread. Disadvantage: a pattern in the list may bias

3 Iteration x_{n+1} = g(x_n) (All boards)

x_{n+1} = ln (x_n + 4) + 1, with x_0 = 2.

Find x_1, x_2 and x_3 to 3 decimal places.

x_1 = 2.7918

x_2 = 2.9157

x_3 = 2.9338

Answer: x_1 = 2.792, x_2 = 2.916, x_3 = 2.934

4 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form y = a(x − p)^2 + q.

The turning point is (−1, 2), so y = a(x + 1)^2 + 2.

The curve passes through (0, 1):

1 = a(1)^2 + 2, so −1 = 1 a

a = −1

Answer: y = −(x + 1)^2 + 2

5 Exact trigonometric values (All boards)

Write down the exact value of sin 240°.

sin 240° has the same size as sin 60°, and the sign comes from the quadrant (CAST).

sin 60° = $\frac{\sqrt{3}}{2}$, so sin 240° = $-\frac{\sqrt{3}}{2}$

Answer: $-\frac{\sqrt{3}}{2}$

6 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−5, 1) lies on the curve y = f(x).

Write down the coordinates of the image of P on

(a) y = −f(x) (b) y = f(−x)

(a) y = −f(x) changes the sign of y, so P maps to (−5, −1).

(b) y = f(−x) changes the sign of x, so P maps to (5, 1).

Answer: (a) (−5, −1) (b) (5, 1)

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7 Particle on a smooth table connected to a hanging particle (All boards)

A 5 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 2 kg particle, as shown. The system is released from rest. Take g = 9.8 m s^{-2}. Find the tension.

Table particle: T = 5 a
Hanging particle: 2 × 9.8 − T = 2 a
Adding: 19.6 = 7 a
Answer: 14 N

8 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-4x}{(x+1)(x-3)}$ in partial fractions.

Write $\frac{-4x}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$
$-4x = A(x-3) + B(x+1)$
$x = -1$: $4 = A(-4)$ so $A = -1$
$x = 3$: $-12 = B(4)$ so $B = -3$
Answer: $\mathbf{\frac{-1}{(x+1)} + \frac{-3}{(x-3)}}$

9 Rough slope: limiting equilibrium or sliding (All boards)

A block is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$. Find the coefficient of friction.

Perpendicular: $R = mg\cos\alpha$
Parallel (limiting): $F = mg\sin\alpha$
$\mu = \frac{F}{R} = \tan\alpha$
Answer: $\mathbf{\frac{4}{3}}$

10 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.5$, H_1: $p < 0.5$.
In a sample, $x = 6$. It is given that $P(X \leq 6) = 0.0577$.
Test H_0 at the 10% significance level.

Compare 0.0577 with 0.1
$0.0577 < 0.1$, so reject H_0
Answer: Reject H_0: evidence that p is less than 0.5