Day 6 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Degrees and radians (All boards)

(a) Convert 210° to radians, giving your answer in terms of π.

(b) Convert $\frac{3\pi}{4}$ radians to degrees.

(a) $210° = 210 \times \frac{\pi}{180} = \frac{7\pi}{6}$

(b) $\frac{3\pi}{4}$ radians $= \frac{3\pi}{4} \times \frac{180}{\pi} = 135°$

Answer: (a) $\frac{7\pi}{6}$ radians (b) 135°

2 Large data set: handling readings (AQA, Edexcel, OCR, MEI, Eduqas)

Seven daily rainfall readings (mm) from the Large Data Set are: 4.4, tr, tr, 1.2, 3.8, 2.2, 0.6.
Here 'tr' means a trace, which is treated as 0.
Find the mean daily rainfall to 3 s.f.

Replace each 'tr' by 0
Total = 12.2
Mean = 12.2 ÷ 7
Answer: 1.74 mm

3 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix}4\\1\\0\end{pmatrix} + t\begin{pmatrix}-2\\1\\-1\end{pmatrix}$.

Find the coordinates of the point on the line where t = 3.

$\mathbf{r} = \begin{pmatrix}4\\1\\0\end{pmatrix} + 3\begin{pmatrix}-2\\1\\-1\end{pmatrix}$

$= \begin{pmatrix}-2\\4\\-3\end{pmatrix}$

Answer: (−2, 4, −3)

4 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 3 \times 2^x - 6$.

Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $2^x \to 0$, so $y \to -6$: asymptote $y = -6$.

$x = 0$: $y = 3 \times 2^0 - 6 = -3$

$y = 0$: $3 \times 2^x = 6$, so $2^x = 2$ and $x = 1$

Answer: y = −6 ; (0, −3), (1, 0)

5 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$.

Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (2)(2) + (3)(-3) + (0)(-2) = -5$

$|\mathbf{a}| = \sqrt{13}$, $|\mathbf{b}| = \sqrt{17}$

$\cos\theta = \frac{-5}{\sqrt{13} \times \sqrt{17}}$, so $\theta = 109.7°$

Answer: 109.7°

6 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 8 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}.
Find (a) its acceleration, (b) its speed after 4 s.

(a) Parallel to slope: $mg\sin\alpha = ma$

$a = 9.8 \times \frac{3}{5}$

(b) $v = u + at = 0 + 5.88 \times 4$

Answer: (a) 5.88 m s^{-2} (b) 23.52 m s^{-1}

7 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that if n^2 is even, then n is even.
Begin: "Assume n is odd, so $n = 2k + 5$ for an integer k."

$n^2 = (2k + 5)^2 = 4k^2 + 20k + 25$

$= 2(2k^2 + 10k + 12) + 1$

This is odd, which contradicts n^2 being even.
So the assumption is false: n must be even.

Answer: Proved

8 Normal approximation to the binomial (CIE)

$X \sim B(200, 0.6)$. Use a normal approximation to find $P(X \geq 117)$, to 3 s.f.

$np = 120$ and $npq = 48$, so use $Y \sim N(120, 48)$
Continuity correction: $P(Y > 116.5)$

$z = (116.5 - 120) \div \sqrt{48}$

Answer: 0.693

9 Two particles over a smooth pulley (All boards)

Two particles of masses 3 kg and 5 kg hang from the ends of a light inextensible string over a smooth pulley and are released from rest. Take $g = 9.8$ m s^{-2}. Find the acceleration and the tension in the string.

5 kg: $5 \times 9.8 - T = 5a$
3 kg: $T - 3 \times 9.8 = 3a$
Adding: $19.6 = 8a$, then $T = 3(9.8 + a)$

Answer: a = 2.45 m s^{-2}, T = 36.75 N

10 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The nth term of a sequence is $u_n = 2n + 1$.
Find $\sum_{r=1}^{10} u_r$.

$u_1 = 3$, $u_{10} = 21$

Sum $= \frac{10}{2}(3 + 21)$

$= 120$

Answer: 120