Day 56 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 5x + 4 and g(x) = x^2 + 2.
Find (a) fg(x), (b) gf(x).

fg(x) = f(x^2 + 2) = 5x^2 + 14
gf(x) = (5x + 4)^2 + 2 = 25x^2 + 40x + 18
Answer: (a) 5x^2 + 14 (b) 25x^2 + 40x + 18

2 Scatter diagrams and correlation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The scatter diagram shows age of a car and value of the car for a group.
Describe the correlation.

The points lie loosely around a line with negative gradient.
As age of a car increases, value of the car tends to decrease.
Answer: weak negative correlation

3 Equation of a tangent or normal (All boards)

Find the equation of the tangent to the curve
$y = 6\sqrt{x} - x$ at the point where x = 4.

dy/dx = −1 + 3/sqrt(x), so the gradient of the curve at x = 4 is $\frac{1}{2}$
At x = 4, y = 8
$y - 8 = \frac{1}{2}(x - 4)$
Answer: $y = \frac{1}{2}x + 6$

4 Constant acceleration: v = u + at (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line has initial speed 11 m s^{-1} and constant acceleration 2 m s^{-2}.
Find its speed after 12 s.

v = u + at
v = 11 + 2 × 12
Answer: 35 m s^{-1}

5 Optimisation (algebraic set-up) (All boards)

The product of x and (48 − x^2) is P = 48x − x^3, x > 0.
Find the maximum value of P.

dP/dx = 48 − 3x^2 = 0, so x^2 = 16 and x = 4
d^2P/dx^2 = −6x < 0, so it is a maximum
P = 48 × 4 − 4^3 = 128
Answer: P = 128

6 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 9 terms of the geometric series 4 + 12 + 36 + ...

a = 4, r = 3, n = 9
$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{4(3^9 - 1)}{3 - 1}$
= 39364
Answer: 39364

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7 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.1$, H_1: $p \neq 0.1$. In a sample, $x = 6$.

Test at the 5% significance level.

Two-tailed test, so compare with $0.05 \div 2 = 0.025$

$P(X \geq 6) = 0.0334$

$0.0334 > 0.025$, so do not reject H_0

Answer: Do not reject H_0 ($P(X \geq 6) = 0.0334$)

8 Integrate using partial fractions (All boards)

$\frac{5x + 13}{(x + 1)(x + 3)} \equiv \frac{A}{x + 1} + \frac{B}{x + 3}$.

Find A and B, and hence find $\int \frac{5x + 13}{(x + 1)(x + 3)}\, dx$.

$5x + 13 \equiv A(x + 3) + B(x + 1)$

$x = -1$ gives $A = 4$; $x = -3$ gives $B = 1$

$\int \left(\frac{4}{x + 1} + \frac{1}{x + 3}\right) dx = 4 \ln|x + 1| + 1 \ln|x + 3| + c$

Answer: A = 4, B = 1; $4 \ln|x + 1| + \ln|x + 3| + c$

9 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 30 kg and length 2 m rests horizontally on supports at A and B. A load of mass 20 kg is placed on the beam 1 m from A. Take $g = 9.8$ m s^{-2}. Find the reaction at B.

Moments about A:

$R_B \times 2 = 30 \times 9.8 \times 1 + 20 \times 9.8 \times 1$

$R_B \times 2 = 490$

Answer: 245 N

10 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{k=1}^{5} 2^{k-1}$.

Geometric series: first term 1, common ratio 2, 5 terms.

$S_5 = \frac{1(2^5 - 1)}{2 - 1}$

$= 31$

Answer: 31