Day 55 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Hypotheses for a binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A company says that the probability a customer buys a ticket is 0.75.
A researcher believes that the probability is lower. Let p be the probability that customer buys a ticket.
Write down suitable hypotheses for a test on p.

H_0 states the claimed value; H_1 matches the researcher's belief.
One-tailed test
Answer: H_0: p = 0.75, H_1: p < 0.75

2 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A line has parametric equations
x = 2t + 1, y = 2t − 5.
Find its equation in the form y = mx + c.

From $x = 2t + 1$: $t = \frac{x - 1}{2}$

$y = 2 \times \frac{x - 1}{2} - 5$

Answer: y = x − 6

3 Tension in a string supporting a hanging mass (All boards)

A particle of mass 10 kg hangs from a light string and is raised vertically with acceleration 1 m s^{-2}.
Taking g = 9.8 m s^{-2}, find the tension.

T − mg = ma
T = 10 × 9.8 + 10 × 1
Answer: 108 N

4 Degrees and radians (All boards)

(a) Convert 120° to radians, giving your answer in terms of π.

(b) Convert $\frac{\pi}{12}$ radians to degrees.

(a) $120° = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$

(b) $\frac{\pi}{12}$ radians $= \frac{\pi}{12} \times \frac{180}{\pi} = 15°$

Answer: (a) $\frac{2\pi}{3}$ radians (b) 15°

5 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-x^3 + 4x^2 + 2x - 2$ is divided by (x − 4).

Remainder = f(4)
= (−64) + 64 + 8 + (−2)
= 6
Answer: 6

6 Equation of a tangent or normal (All boards)

Find the equation of the tangent to the curve
$y = 3x^3 + x^2 - 2$ at the point where x = 1.

dy/dx = 9x^2 + 2x, so the gradient of the curve at x = 1 is 11
At x = 1, y = 2
y − 2 = 11(x − 1)
Answer: y = 11x − 9

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7 Normal approximation to the binomial (CIE)

X ~ B(80, 0.3). Use a normal approximation to find P(X ≥ 22), to 3 s.f.

np = 24 and npq = 16.8, so use Y ~ N(24, 16.8)
Continuity correction: P(Y > 21.5)
$z = (21.5 - 24) \div \sqrt{16.8}$
Answer: 0.729

8 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 2x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -\frac{1}{2}$, $y = 2x$

$= 1 + (-\frac{1}{2})(2x) + \frac{(-\frac{1}{2})((-\frac{3}{2}))}{2}(2x)^2$

$= 1 - x + \frac{3}{2}x^2$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 - x + \frac{3}{2}x^2$; $|x| < \frac{1}{2}$

9 Product rule (All boards)

Use the product rule to differentiate
$y = x^3 \cos 4x$.

$u = x^3$, $v = \cos 4x$
$u' = 3x^2$, $v' = -4 \sin 4x$
$dy/dx = u'v + uv'$, which simplifies to the answer below
Answer: $dy/dx = x^2(3 \cos 4x - 4x \sin 4x)$

10 Maximum friction on a horizontal surface (All boards)

A box of mass 4 kg rests on rough horizontal ground. The coefficient of friction is 0.4.
Taking g = 9.8 m s^{-2}, find the maximum friction force that can act on the box.

$R = mg = 4 \times 9.8 = 39.2$
$F_{max} = \mu R = 0.4 \times 39.2$
Answer: 15.68 N