Day 53 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Exact trigonometric values (All boards)

Write down the exact value of cos 315°.

cos 315° has the same size as cos 45°, and the sign comes from the quadrant (CAST).

cos 45° = $\frac{\sqrt{2}}{2}$, so cos 315° = $\frac{\sqrt{2}}{2}$

Answer: $\frac{\sqrt{2}}{2}$

2 Tension in a string supporting a hanging mass (All boards)

A particle of mass 12 kg hangs from a light string and is raised at constant speed. Taking g = 9.8 m s^{-2}, find the tension in the string.

No acceleration, so T = mg

T = 12 × 9.8

Answer: 117.6 N

3 Combined mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mean of 15 ages of the men is 68. The mean of 15 ages of the women is 63.
Find the mean of all 30 values.

Total of first group = 15 × 68 = 1020

Total of second group = 15 × 63 = 945

Mean = 1965 ÷ 30

Answer: 65.5

4 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has common ratio $\frac{2}{3}$ and sum to infinity 18. Find the first term.

$S_\infty = \frac{a}{1-r}$, so $18 = \frac{a}{1-\frac{2}{3}}$

$a = 18 \times \frac{1}{3}$

Answer: a = 6

5 Optimisation (algebraic set-up) (All boards)

A rectangle has perimeter 50 cm.
Find the maximum possible area of the rectangle and the dimensions that give it.

Sides x and 25 − x, so $A = x(25-x) = 25x - x^2$

$dA/dx = 25 - 2x = 0$, so $x = \frac{25}{2}$

$d^2A/dx^2 = -2 < 0$, so it is a maximum; $A = \frac{25}{2} \times \frac{25}{2}$

Answer: A = $\frac{625}{4}$ cm², a $\frac{25}{2}$ cm square

6 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{3}{x-2} + 2$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: x = 2 and y = 2.

y = 0: $\frac{3}{x-2} = -2$, so $x = \frac{1}{2}$

x = 0: $y = \frac{3}{-2} + 2 = \frac{1}{2}$

Answer: x = 2, y = 2 ; ($\frac{1}{2}$, 0), (0, $\frac{1}{2}$)

7 Conditional probability: two-way tables (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Of 90 people, 58 are girls. 35 of the girls and 21 of the boys play an instrument.
One person is chosen at random.
Given that the person is a girl, find the probability that they play an instrument.

There are 58 girls
P = 35 ÷ 58
Answer: $\mathbf{\frac{35}{58}}$

8 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation xy = 2.
Find dy/dx in terms of x and y, and hence the gradient at (2, 1).

Differentiate both sides with respect to x (using dy/dx for y terms)

dy/dx = $-\frac{y}{x}$

At (2, 1): dy/dx = $-\frac{1}{2}$

Answer: dy/dx = $-\frac{y}{x}$; gradient $-\frac{1}{2}$

9 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O at constant speed 4 m s^{-1}.
At that instant car Q starts from rest at O with constant acceleration 1 m s^{-2} in the same direction. Find the time when Q catches up P.

Distances equal: $4t = \frac{1}{2} \times 1 \times t^2$

$t(4 - 0.5t) = 0$, so $t = 0$ or $t = 4 \div 0.5$

Answer: 8 s

10 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = −5x − 6.
Find $f^{-1}(x)$.

y = −5x − 6, so y + 6 = −5x

$x = \frac{y + 6}{-5}$

Answer: $\mathbf{f^{-1}(x) = \frac{x + 6}{-5}}$