Day 52 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $2x^2 + 12x + 14$ in the form $a(x + p)^2 + q$.

Take out a factor 2: $2(x^2 + 6x + 7)$

$x + 6 = (x + 3)^2 - 9$

$2[(x + 3)^2 - 9] + 14 = 2(x + 3)^2 - 4$

Answer: $2(x + 3)^2 - 4$

2 Tension in a string supporting a hanging mass (All boards)

A particle of mass 10 kg hangs from a light string and is raised vertically with acceleration 0.5 m s^{-2}.
Taking g = 9.8 m s^{-2}, find the tension.

$T - mg = ma$

$T = 10 \times 9.8 + 10 \times 0.5$

Answer: 103 N

3 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 10 and 20 minutes.

Frequency = frequency density × class width

$10 \leq t < 15$: $3 \times 5 = 15$

$15 \leq t < 20$: $3 \times 5 = 15$

Total = 15 + 15

Answer: 30

4 Product rule (All boards)

Use the product rule to differentiate
$y = x^1 \ln x$.

$u = x^1$, $v = \ln x$

$u' = 1$, $v' = \frac{1}{x}$

$dy/dx = u'v + uv'$, which simplifies to the answer below

Answer: $dy/dx = \ln x + 1$

5 Normal approximation to the binomial (CIE)

X ~ B(40, 0.6). Use a normal approximation to find $P(X \geq 22)$, to 3 s.f.

$np = 24$ and $npq = 9.6$, so use Y ~ N(24, 9.6)

Continuity correction: $P(Y > 21.5)$

$z = (21.5 - 24) \div \sqrt{9.6}$

Answer: 0.790

6 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ab^x$. A graph of $\log_{10} y$ against x is a straight line with gradient 0.7 and intercept 1.5.
Find a and b to 3 significant figures.

$\log y = \log a + x \log b$

$\log b = 0.7$ so $b = 10^{0.7} = 5.01$

$\log a = 1.5$ so $a = 10^{1.5} = 31.6$

Answer: a = 31.6, b = 5.01

7 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{x + 3} + 2$.

State the largest possible domain and the range of f.

$x + 3 \geq 0$, so $x \geq -3$.

$\sqrt{...} \geq 0$, so $f(x) \geq 2$.

Answer: Domain $x \geq -3$, range $f(x) \geq 2$

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8 Work done and kinetic energy (CIE)

A constant force of 80 N pulls a crate 10 m along rough horizontal ground against a constant frictional force of 10 N. Find (a) the work done by the pulling force, (b) the work done against friction.

(a) Work done = force × distance = 80 × 10

(b) Work done against friction = 10 × 10

Answer: (a) 800 J (b) 100 J

9 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of
(a) arctan(−1) (b) arccos(1)

(a) $y = \arctan(-1)$ means $\tan y = -1$ with $-\frac{\pi}{2} < y < \frac{\pi}{2}$: $y = -\frac{\pi}{4}$

(b) $y = \arccos(1)$ means $\cos y = 1$ with $0 \leq y \leq \pi$: $y = 0$

Answer: (a) $-\frac{\pi}{4}$ (b) 0

10 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{1}{x-1} + 1$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: $x = 1$ and $y = 1$.

$y = 0$: $\frac{1}{x-1} = -1$, so $x = 0$

$x = 0$: $y = \frac{1}{-1} + 1 = 0$

Answer: x = 1, y = 1 ; (0, 0), (0, 0)