Day 51 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x + 3)^2 + (y - 1)^2 = 64$.
Determine whether the point P(7, 8) is inside, outside or on the circle.

Distance^2 from the centre $(-3, 1)$ to P:
$(7 - (-3))^2 + (8 - 1)^2 = 100 + 49 = 149$
$r^2 = 64$, and $149 > 64$.
Answer: P is outside the circle

2 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 2$.
Use the Newton-Raphson method with $x_0 = 3$ to find x_1 and x_2,
giving your answer to 3 decimal places.

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 3 - \frac{7.000}{6.000} = 1.8333$$

$$x_2 = 1.8333 - \frac{1.361}{3.667} = 1.4621$$

Answer: $x_1 = 1.833$, $x_2 = 1.462$

3 Binomial: P(X = k) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a seed grows is 0.5. 20 seeds are planted. X is the number that grow.
Find P(X = 10), giving your answer to 3 s.f.

$P(X = 10) = {}^{20}C_{10} \times 0.5^{10} \times 0.5^{10}$
$= 184756 \times 0.5^{10} \times 0.5^{10}$
Answer: 0.176

4 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of five consecutive integers is a multiple of 5.

Let the integers be n, n + 1, ..., n + 4.
Sum = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10
= 5(n + 2), which is a multiple of 5.
Answer: Shown

5 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\sin A = \frac{24}{25}$.
Find the exact values of sin 2A and cos 2A.

$$\cos A = \sqrt{1 - \sin^2 A} = \frac{7}{25}$$

$$\sin 2A = 2 \sin A \cos A = 2 \times \frac{24}{25} \times \frac{7}{25} = \frac{336}{625}$$

$$\cos 2A = \cos^2 A - \sin^2 A = \frac{49}{625} - \frac{576}{625} = -\frac{527}{625}$$

Answer: $\sin 2A = \frac{336}{625}$, $\cos 2A = -\frac{527}{625}$

6 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and moves in a straight line with constant acceleration 4 m s^{-2}.
It travels 18 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$

$18 = \frac{1}{2} \times 4 \times t^2$

$t^2 = 9$

Answer: 3 s

7 Normal approximation to the binomial (CIE)

X ~ B(150, 0.3). Use a normal approximation to find P(X ≤ 51), to 3 s.f.

np = 45 and npq = 31.5, so use Y ~ N(45, 31.5)
Continuity correction: P(Y < 51.5)
$z = (51.5 - 45) \div \sqrt{31.5}$
Answer: 0.877

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 4x + k is a tangent to the curve
$y = x^2 + 8x - 4$.
Find the value of k.

$x^2 + 8x - 4 = 4x + k$

$x^2 + 4x + (-4 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(-4 - k) = 0$

Answer: k = −8

9 Driving force and resistance (All boards)

A car of mass 1200 kg is driven along a straight horizontal road by a driving force of 1600 N against a constant resistance of 400 N.
Find the acceleration of the car.

Resultant force = 1600 − 400 = 1200 N
F = ma: 1200 = 1200 a
Answer: 1 m s^{-2}

10 Solve a logarithmic equation (one log) (All boards)

Solve $\log_x 27 = 3$.

$x^3 = 27$
x = 3
Answer: x = 3