Day 50 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 6x - 10y - 30 = 0$.
Find the centre and the radius of the circle.

Complete the square in x and in y:
$(x - 3)^2 + (y - 5)^2 = 64$
Centre (3, 5), radius = 8
Answer: Centre (3, 5), radius 8

2 Connected rates of change (All boards)

A circular ripple has radius r cm, increasing at 0.5 cm/s.
Find the rate at which the area is increasing when r = 3, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$
$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi \times 3 \times 0.5$
Answer: 3π cm^2/s

3 Binomial: mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(25, 0.4). Find E(X) and Var(X).

E(X) = np = 25 × 0.4 = 10
Var(X) = npq = 25 × 0.4 × 0.6
Answer: E(X) = 10, Var(X) = 6

4 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A body moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 + t^2 - 5t + 2$.
Find its velocity when t = 4.

$v = \frac{ds}{dt} = 6t^2 + 2t - 5$
When t = 4: $v = 6 \times 4^2 + 2 \times 4 - 5$
Answer: 99 m s^{-1}

5 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form px + qy = r with integers p, q and r, (b) the length AB.

Gradient = $\frac{-1 - 0}{0 - 1} = 1$
Using y = mx + c with c = −1: y = x − 1, so x − y = 1
$AB^2 = 1^2 + 1^2 = 2$
Answer: (a) x − y = 1 (b) $\sqrt{2}$

6 Resolving a force into components (All boards)

A rope pulls a box with a force of 16 N at an angle of 30° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 16 cos 30°
Vertical = 16 sin 30°
Answer: horizontal 13.9 N, vertical 8.00 N

7 Rearrange and iterate (All boards)

(a) Show that $x^3 - 4x - 3 = 0$ can be rearranged to $x = (4x + 3)^{\frac{1}{3}}$.
(b) Use x_{n+1} = this formula with $x_0 = 2$ to find x_1, x_2 to 3 d.p.

(a) $x^3 = 4x + 3$, so $x = \sqrt[3]{(4x + 3)}$
(b) $x_1 = 2.2240$, $x_2 = 2.2828$
Answer: (a) shown (b) $x_1 = 2.224$, $x_2 = 2.283$

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8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 5^2)$. H_0: $\mu = 170$, H_1: $\mu > 170$.

A sample of 36 has mean $\bar{x} = 172.25$.
Test at the 5% level. Critical value: $z = 1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (172.25 - 170) \div (5 \div 6)$
$z = 2.70$
2.70 is inside the critical region, so reject H_0
Answer: $z = 2.70$; reject H_0

9 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \cos^2 3x \, dx$.

$\cos 2\theta = 2\cos^2\theta - 1$, so $\cos^2 3x = \frac{1}{2}(1 + \cos 6x)$

Integrate term by term

Answer: $\frac{x}{2} + \frac{\sin 6x}{12} + c$

10 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, where $f(x)$ is a cubic. Find $f(x)$ in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -4$, -2 and 1.
$f(x) = k(x + 4)(x + 2)(x - 1)$
At (0, 8): $k \times 4 \times 2 \times (-1) = 8$
$k = -1$
Answer: $f(x) = -(x + 4)(x + 2)(x - 1)$