Day 5 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In an arithmetic sequence $u_5 = 25$ and $u_7 = 33$.

Find the first term and the common difference.

Subtract: $(7 - 5)d = 33 - 25$

$2d = 8$, so $d = 4$

$a = u_5 - 4\,d = 25 - 16 = 9$

Answer: a = 9, d = 4

2 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) $\arcsin(-\frac{1}{2})$ (b) $\arcsin(1)$

(a) $y = \arcsin(-\frac{1}{2})$ means $\sin y = -\frac{1}{2}$ with $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$: $y = -\frac{\pi}{6}$

(b) $y = \arcsin(1)$ means $\sin y = 1$ with $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$: $y = \frac{\pi}{2}$

Answer: (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

3 Discrete random variable: E(X) and Var(X) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has this distribution.

x: 0, 6, 7, 8

P(X = x): 0.4, 0.2, 0.3, 0.1

Find E(X) and Var(X).

$E(X) = \Sigma xp = 0\times0.4 + 6\times0.2 + 7\times0.3 + 8\times0.1 = 4.1$

$E(X^2) = \Sigma x^2 p = 28.3$

$\text{Var}(X) = 28.3 - 4.1^2$

Answer: E(X) = 4.1, Var(X) = 11.49

4 Power of an engine: driving force (CIE)

A train's engine works at a constant rate of 22.5 kW. Find the driving force when the vehicle is moving at 15 m s^{-1}.

$P = Fv$

$22500 = F \times 15$

Answer: 1500 N

5 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = 3t$, $y = 3t^2 + 1$.

Find the gradient at the point where $t = 3$.

$dx/dt = 3$, $dy/dt = 6t$

$dy/dx = \frac{6t}{3}$

At $t = 3$: $dy/dx = 6$

Answer: 6

6 Particle held by a string and a horizontal force (All boards)

A ball of weight 12 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan\theta = \frac{4}{3}$. Find P and the tension T N.

Vertically: $T\cos\theta = 12$

Horizontally: $T\sin\theta = P$

Dividing: $P = 12\tan\theta$; $T = \sqrt{12^2 + P^2}$

Answer: P = 16, T = 20

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7 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-10x + 12}{(x + 3)(x - 4)}$ in partial fractions.

Write $\frac{-10x + 12}{(x + 3)(x - 4)} = \frac{A}{(x + 3)} + \frac{B}{(x - 4)}$

$-10x + 12 = A(x - 4) + B(x + 3)$

$x = -3$: $42 = A(-7)$ so $A = -6$

$x = 4$: $-28 = B(7)$ so $B = -4$

Answer: $\mathbf{\frac{-6}{(x + 3)} + \frac{-4}{(x - 4)}}$

8 Parallel vectors (All boards)

$\mathbf{a} = 4\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 12\mathbf{i} + k\mathbf{j}$, where k is a constant.
Given that a and b are parallel, find the value of k.

Parallel: $\mathbf{b} = \lambda\mathbf{a}$, so $12 = \lambda \times 4$, giving $\lambda = 3$

$k = \lambda \times 2 = 6$

Answer: k = 6

9 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of the squares of two integers that differ by 7 is always odd.

Let the integers be n and n + 7.

$n^2 + (n + 7)^2 = 2n^2 + 14n + 49$

$= 2(n^2 + 7n) + 49$

$2(n^2 + 7n)$ is even and 49 is odd, so the sum is odd.

Answer: Shown

10 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.5. If so, the probability that the patient recovers is 0.5; otherwise it is 0.3.
Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.5 × 0.5 = 0.25; P(not first and second) = 0.5 × 0.3 = 0.15

P(second) = 0.25 + 0.15 = 0.4

P = 0.25 ÷ 0.4

Answer: $\mathbf{\frac{5}{8} = 0.625}$