Day 49 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x - 2 \geq -4x + 9$.

Collect x terms: $6x \geq 11$

Divide by 6: $x \geq \frac{11}{6}$

Answer: $x \geq \frac{11}{6}$

2 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(x^2 + 1)$ for $x = 0, 1, 2, 3$.
(a) Find the value of k.
(b) Find $P(X \geq 2)$.

(a) Probabilities sum to 1: $k(1 + 2 + 5 + 10) = 1$

$k = \frac{1}{18}$

(b) $P(X \geq 2) = \frac{5}{18} + \frac{10}{18}$

Answer: (a) $k = \frac{1}{18}$ (b) $\frac{5}{6}$

3 Solve trig equation in radians (calculator) (All boards)

Solve $4 \cos x = 2$ for $0 \leq x \leq 2\pi$.
Give your answers to 3 significant figures.

$\cos x = \frac{1}{2}$

Reference angle = $\cos^{-1}(0.5000) = 1.047$ radians
cos is positive in the first and fourth quadrants.
$x = 1.05, 5.24$
Answer: x = 1.05, 5.24

4 Power of an engine: driving force (CIE)

A car's engine works at a constant rate of 30 kW. Find the driving force when the vehicle is moving at 30 m s^{-1}.

$P = Fv$
$30000 = F \times 30$
Answer: 1000 N

5 Solve a logarithmic equation (two logs) (All boards)

Solve $\ln(x + 3) - \ln x = \ln 4$.

$\ln \frac{x + 3}{x} = \ln 4$, so $\frac{x + 3}{x} = 4$
$x + 3 = 4x$, so $3 = 3x$
$x = 1$
Answer: x = 1

6 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = k(x + 3)$ for $0 \leq x \leq 3$, and 0 otherwise.
Find the value of k.

$\int$ from 0 to 3 of $k(x + 3)\,dx = 1$

$k[\frac{x^2}{2} + 3x]$ from 0 to 3 $= k(\frac{27}{2}) = 1$

Answer: $k = \frac{2}{27}$

7 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 2x)^{-1}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -1$, $y = -2x$

$= 1 + (-1)(-2x) + \frac{(-1)((-2))}{2}(-2x)^2$

$= 1 + 2x + 4x^2$

Valid when $|-2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 + 2x + 4x^2$; $|x| < \frac{1}{2}$

8 Car towing a caravan (All boards)

A car of mass 1500 kg tows a caravan of mass 400 kg on a level road with acceleration 0.25 m s^{-2}. Resistances: 200 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 200 - 100 = 1900 \times 0.25$
Caravan: $T - 100 = 400 \times 0.25$
Answer: D = 775 N, T = 200 N

9 Connected rates of change (All boards)

The side x cm of a cube increases at 0.25 cm/s.
Find the rate of increase of the volume when x = 3.

$V = x^3$, so $\frac{dV}{dx} = 3x^2$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3 \times 3^2 \times 0.25$

Answer: 6.75 cm^3/s

10 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $x^3 - 2x^2 - 3x - 5$ is divided by $(x - 3)$.

Remainder = f(3)
$= 27 + (-18) + (-9) + (-5)$
$= -5$
Answer: -5