Day 48 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $2\,e^{4x}$

(b) $2\,e^{x} + 4 \ln x$

(a) $dy/dx = 8\,e^{4x}$

(b) $dy/dx = 2\,e^{x} + \frac{4}{x}$

Answer: (a) $8\,e^{4x}$ (b) $2\,e^{x} + \frac{4}{x}$

2 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points P and Q, 182 m apart, and cycle directly towards each other at constant speeds of 7 m s^{-1} and 6 m s^{-1}. Find the time until they meet.

Closing speed = 7 + 6 = 13 m s^{-1}

Time = $\frac{182}{13}$

Answer: 14 s

3 Definite integrals (exact) (All boards)

Find the exact value of

$\int_0^1 5\,e^{4x}\,dx$.

$[\frac{5}{4}\,e^{4x}]$ from 0 to 1

$= \frac{5}{4}\,e^{4} - \frac{5}{4}$

$= \frac{5(e^{4} - 1)}{4}$

Answer: $\frac{5(e^{4} - 1)}{4}$

4 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(1, 3) lies on the curve y = f(x).

Write down the coordinates of the image of P on

(a) y = f(x + 1) (b) y = f(x − 1)

(a) y = f(x + 1) subtracts 1 from x, so P maps to (0, 3).

(b) y = f(x − 1) adds 1 to x, so P maps to (2, 3).

Answer: (a) (0, 3) (b) (2, 3)

5 Friction: constant speed, find the coefficient (All boards)

A block of mass 4 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 9.8 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the block and ground.

R = 4 × 9.8 = 39.2

Constant speed: F = P = 9.8

F = μR: 9.8 = μ × 39.2

Answer: 0.25

6 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 2 red and 6 blue sweets. Two are taken at random without replacement.

Find the probability that at least one of them is red.

P(no red) = P(BB) = $\frac{6}{8} \times \frac{5}{7} = \frac{15}{28}$

P(at least one red) = $1 - \frac{15}{28}$

Answer: $\frac{13}{28}$

7 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 2x)^{-2}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + ...$ with $n = -2$, $y = -2x$

$= 1 + (-2)(-2x) + \frac{(-2)((-3))}{2}(-2x)^2$

$= 1 + 4x + 12x^2$

Valid when $|-2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 + 4x + 12x^2$; $|x| < \frac{1}{2}$

8 Solve trig equation in radians (exact) (All boards)

Solve $\cos x = -\frac{\sqrt{2}}{2}$ for $0 \leq x \leq 2\pi$.

Give your answers in terms of π.

Reference angle: $\frac{\pi}{4}$

cos is negative in the second and third quadrants.

$x = \frac{3\pi}{4}, \frac{5\pi}{4}$

Answer: $x = \frac{3\pi}{4}, \frac{5\pi}{4}$

9 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{2}{9}x$ for $0 \leq x \leq 3$, and 0 otherwise.

Find $P(X < 1)$.

$P(X < 1) = \int$ from 0 to 1 of $\frac{2}{9}x\,dx$

$= [\frac{2}{18}x^2]$ from 0 to 1

Answer: $\frac{1}{9}$

10 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 - 6x^2 - 36x + 80$

(a) Show that $(x + 4)$ is a factor of f(x).

(b) Factorise f(x) completely.

$f(-4) = (-128) + (-96) + 144 + 80 = 0$, so $(x + 4)$ is a factor.

$f(x) = (x + 4)(2x^2 - 14x + 20)$

$2x^2 - 14x + 20 = 2(x - 2)(x - 5)$

Answer: (b) $f(x) = 2(x + 4)(x - 2)(x - 5)$