Day 46 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Regression: interpreting coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

A model relates the temperature, T °C, of a cooling drink to the time, t minutes, since pouring by $T = 77 - 2.5t$.
Interpret the value 2.5 in this context.

The gradient is the change in T for each 1 unit of t.
For each extra minute, T decreases by 2.5 °C.
Answer: each extra minute, T decreases by 2.5 °C

2 Simplify using identities (All boards)

Simplify $\tan x \cos x$.

$\tan x = \frac{\sin x}{\cos x}$, so $\tan x \cos x = \sin x$
Answer: sin x

3 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity
$(3\mathbf{i} - 2\mathbf{j})$ m s^{-1}. It is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m at t = 0. Find the time when its position vector is $(7\mathbf{i} - 3\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
$(7\mathbf{i} - 3\mathbf{j}) = (\mathbf{i} + \mathbf{j}) + t(3\mathbf{i} - 2\mathbf{j})$
Equating i components: $7 = 1 + 3t$
Answer: 2 s

4 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form px + qy = r with integers p, q and r, (b) the length AB.

Gradient $= \frac{-6 - 0}{0 - (-1)} = -6$
Using y = mx + c with c = −6: $y = -6x - 6$, so $6x + y = -6$
$AB^2 = 1^2 + 6^2 = 37$
Answer: (a) 6x + y = −6 (b) $\sqrt{37}$

5 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 - 2x - 5$.
Use the Newton-Raphson method with $x_0 = 2$ to find x_1 and x_2,
giving your answer to 3 decimal places.

$f'(x) = 3x^2 - 2$
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
$x_1 = 2 - \frac{-1.000}{10.00} = 2.1000$
$x_2 = 2.1000 - \frac{0.06100}{11.23} = 2.0946$
Answer: $x_1 = 2.100$, $x_2 = 2.095$

6 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 6^2)$. H_0: $\mu = 35$, H_1: $\mu \neq 35$.
A sample of 64 has mean $\bar{x} = 36.2$.
Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (36.2 - 35) \div (6 \div 8)$
$z = 1.60$
1.60 is outside the critical region, so do not reject H_0
Answer: z = 1.60; do not reject H_0

7 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line increases its speed from 4 m s^{-1} to 11 m s^{-1} with constant acceleration 0.5 m s^{-2}. Find the time taken.

$v = u + at$
$11 = 4 + 0.5\,t$
Answer: 14 s

8 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{x-5}{(x+1)^2}$ in the form $\frac{A}{(x+1)} + \frac{B}{(x+1)^2}$.

$x - 5 = A(x + 1) + B$
Coefficient of x: $A = 1$
Constant: $-5 = -1 + B$, so $B = -6$
Answer: A = 1, B = −6

9 Optimisation (algebraic set-up) (All boards)

A farmer has 24 m of fencing for three sides of a rectangular pen; the fourth side is a wall.
Find the maximum area of the pen.

Two sides of length x m: $A = x(24 - 2x) = 24x - 2x^2$
$dA/dx = 24 - 4x = 0$, so $x = 6$
$d^2A/dx^2 = -4 < 0$ (maximum); $A = 6 \times 12$
Answer: 72 m^2

10 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2 \times 5^x - 250$.
Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $5^x \to 0$, so $y \to -250$: asymptote $y = -250$.
$x = 0$: $y = 2 \times 5^0 - 250 = -248$
$y = 0$: $2 \times 5^x = 250$, so $5^x = 125$ and $x = 3$
Answer: y = −250 ; (0, −248), (3, 0)