Day 45 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = \frac{1}{2}$, $y = -2x$

$= 1 + \frac{1}{2}(-2x) + \frac{\frac{1}{2}((-\frac{1}{2}))}{2}(-2x)^2$

$= 1 - x - \frac{1}{2}x^2$

Valid when $|-2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 - x - \frac{1}{2}x^2$; $|x| < \frac{1}{2}$

2 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = (x^2 + 2)^2$
at the point where x = 0.

Use the chain rule: $dy/dx = 2(x^2 + 2)^1 \times 2x$
At x = 0: dy/dx = 0
Answer: 0

3 Evaluating fractional and negative indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $25^{\frac{3}{2}}$.

$25^{\frac{3}{2}} = (25^{\frac{1}{2}})^3$

$25^{\frac{1}{2}} = 5$

$(5)^3 = 125$

Answer: 125

4 Compound growth: find n (All boards)

£2000 invested at 4% compound interest per year.
After how many whole years will it first exceed £6000?

$2000 \times 1.04^n > 6000$, so $1.04^n > 3$

$n > \frac{\log 3}{\log 1.04} = 28.01$

The least whole number of years is 29
Answer: 29 years

5 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that exactly one of R and L happens.

Exactly one: (R and L′) or (R′ and L)
$= 0.8 \times 0.4 + 0.2 \times 0.8$
Answer: 0.48

6 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a runner.
(a) Find the acceleration in the first 5 s.
(b) Find the deceleration in the last 2 s.
(c) Find the total distance travelled.

(a) $a = \frac{16 - 4}{5}$

(b) $a = \frac{16 - 0}{2}$, a deceleration of 8

(c) area $= \frac{1}{2}(4 + 16) \times 5 + 16 \times 8 + \frac{1}{2} \times 2 \times 16$

Answer: (a) 2.4 m s^{-2} (b) 8 m s^{-2} (c) 194 m

7 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x + 2)^2 + (y + 1)^2 = 25$.
Determine whether the point P(8, 5) is inside, outside or on the circle.

Distance^2 from the centre $(-2, -1)$ to P:
$(8 - (-2))^2 + (5 - (-1))^2 = 100 + 36 = 136$
$r^2 = 25$, and $136 > 25$.
Answer: P is outside the circle

8 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x = 4t^2 + 3$, $y = 3t^3 - t$.
Find dy/dx in terms of t, and its value when t = 2.

$dx/dt = 8t$, $dy/dt = 9t^2 - 1$

$dy/dx = \frac{dy}{dt \div dx/dt} = \frac{9t^2 - 1}{8t}$

At t = 2: $dy/dx = \frac{35}{16}$

Answer: $dy/dx = \frac{9t^2 - 1}{8t}$; $\frac{35}{16}$

9 Resultant of perpendicular forces (All boards)

Two forces of magnitudes 6 N and 8 N act on a small ring at right angles to each other. Find (a) the magnitude of the resultant, (b) the angle it makes with the 6 N force, to 1 d.p.

(a) $R^2 = 6^2 + 8^2 = 100$
$R = \sqrt{100}$
(b) $\tan\theta = \frac{8}{6}$

Answer: (a) 10 N (b) 53.1°

10 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.2. If so, the probability that the patient recovers is 0.7; otherwise it is 0.5. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.2 × 0.7 = 0.14; P(not first and second) = 0.8 × 0.5 = 0.4
P(second) = 0.14 + 0.4 = 0.54
P = 0.14 ÷ 0.54

Answer: $\frac{7}{27}$ = 0.259