Day 44 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -7\mathbf{i} + 24\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{7^2 + 24^2} = 25$

(b) Acute angle = $\tan^{-1}(\frac{24}{7}) = 73.7°$

p is in the second quadrant, so the angle is 106.3°

Answer: (a) 25 (b) 106.3°

2 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins 11, 17, 23, 29, ...
Find (a) an expression for the nth term,
(b) the 36th term.

$a = 11$, $d = 6$

$u_n = a + (n - 1)d = 11 + (n - 1)(6) = 6n + 5$

$u_{36} = 221$

Answer: (a) $6n + 5$ (b) 221

3 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is $(3t + 5)$ m s^{-2}.
When t = 0 its velocity is 2 m s^{-1}.
Find its velocity when t = 6.

$v = \int a\,dt = 1.5t^2 + 5t + c$

$t = 0$, $v = 2$ gives $c = 2$

When $t = 6$: $v = 54 + 30 + 2$

Answer: 86 m s^{-1}

4 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

12 pairs of data on daily mean temperature and sunshine hours have $r = 0.35$.
Test, at the 5% significance level, for evidence of positive correlation. The critical value is 0.4973.

$H_0: \rho = 0$, $H_1: \rho > 0$

$r = 0.35$ is not beyond the critical value 0.4973

Do not reject H_0

Answer: Do not reject H_0: there is not enough evidence of positive correlation

5 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t$, $y = \frac{6}{t}$, $t \neq 0$.

Find a Cartesian equation of the curve.

$t = x$

$y = \frac{6}{t} = \frac{6}{x}$

Answer: $y = \frac{6}{x}$

6 Connected rates of change (All boards)

Water fills a tank with a square base of side 10 m.
The depth h m increases at 0.5 m/min. Find the rate of increase of volume.

$V = 10^2h = 100h$, so $\frac{dV}{dh} = 100$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} = 100 \times 0.5$

Answer: 50 m³/min

7 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form px + qy = r with integers p, q and r, (b) the length AB.

Gradient = $\frac{4-0}{0-(-2)} = 2$

Using y = mx + c with c = 4: y = 2x + 4, so $2x - y = -4$

$AB^2 = 2^2 + 4^2 = 20$

Answer: (a) $2x - y = -4$ (b) $2\sqrt{5}$

8 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 13 kg rests on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg\sin\alpha = 13 \times 9.8 \times \frac{3}{5}$

Perpendicular: $R = mg\cos\alpha = 13 \times 9.8 \times \frac{4}{5}$

Answer: P = 76.44, R = 101.92

9 Combined mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mean of 40 times (seconds) of the Class A is 79. The mean of 40 times (seconds) of the Class B is 75.
Find the mean of all 80 values.

Total of first group = 40 × 79 = 3160
Total of second group = 40 × 75 = 3000
Mean = 6160 ÷ 80
Answer: 77

10 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \sin^2 3x\,dx$.

$\cos 2\theta = 1 - 2\sin^2\theta$, so $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$

Integrate term by term

Answer: $\frac{x}{2} - \frac{\sin 6x}{12} + c$