Day 43 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -2x + 1$.
Find the equation of the line perpendicular to l that passes through (1, 1).

Gradient of l is -2, so the perpendicular gradient is $\frac{1}{2}$.

$y - 1 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x + \frac{1}{2}$

Answer: $y = \frac{1}{2}x + \frac{1}{2}$

2 Constant acceleration: distance from average speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train moving in a straight line with constant acceleration increases its speed from 4 m s^{-1} to 9 m s^{-1} in 6 s.
Find the distance it travels in this time.

$s = \frac{(u + v)}{2} t$

$s = \frac{(4 + 9)}{2} \times 6$

Answer: 39 m

3 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = 2x(3 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where $x = 0$ and $x = 3$

Area $= \int_0^3 2(3x - x^2)\,dx = [2(\frac{3x^2}{2} - \frac{x^3}{3})]$ from 0 to 3

$= 2(\frac{3^3}{2} - \frac{3^3}{3})$

$= 9$

Answer: 9

4 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.
Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is (3, 2), so $y = a(x - 3)^2 + 2$.
The curve passes through (0, -16):
$-16 = a(-3)^2 + 2$, so $-18 = 9a$
$a = -2$

Answer: $y = -2(x - 3)^2 + 2$

5 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix}1\\4\\3\end{pmatrix} + t\begin{pmatrix}2\\-2\\2\end{pmatrix}$.
Find the coordinates of the point on the line where $t = 2$.

$\mathbf{r} = \begin{pmatrix}1\\4\\3\end{pmatrix} + 2\begin{pmatrix}2\\-2\\2\end{pmatrix}$

$= \begin{pmatrix}5\\0\\7\end{pmatrix}$

Answer: (5, 0, 7)

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6 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + (y + 1)^2 = 25$.
Determine whether the point P$(-6, -2)$ is inside, outside or on the circle.

Distance^2 from the centre $(0, -1)$ to P:
$(-6 - 0)^2 + (-2 - (-1))^2 = 36 + 1 = 37$
$r^2 = 25$, and $37 > 25$.
Answer: P is outside the circle

7 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \sin^2 2x \, dx$.

$\cos 2\theta = 1 - 2\sin^2\theta$, so $\sin^2 2x = \frac{1}{2}(1 - \cos 4x)$

Integrate term by term

Answer: $\frac{x}{2} - \frac{\sin 4x}{8} + c$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.6. If so, the probability that an item is faulty is 0.3; otherwise it is 0.2. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.6 × 0.3 = 0.18; P(not first and second) = 0.4 × 0.2 = 0.08
P(second) = 0.18 + 0.08 = 0.26
P = 0.18 ÷ 0.26

Answer: $\frac{9}{13} = 0.692$

9 Two particles over a smooth pulley (All boards)

Two particles of masses 6 kg and 9 kg hang from the ends of a light inextensible string over a smooth pulley and are released from rest. Take g = 9.8 m s^{-2}. Find the acceleration and the tension in the string.

9 kg: $9 \times 9.8 - T = 9a$
6 kg: $T - 6 \times 9.8 = 6a$
Adding: $29.4 = 15a$, then $T = 6(9.8 + a)$

Answer: a = 1.96 m s^{-2}, T = 70.56 N

10 Mean from a frequency table (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises the number of pets owned by 50 people.
Pets, x: 0, 1, 2, 3, 4
Frequency, f: 9, 22, 12, 3, 4
Find the mean.

$\Sigma fx = 9\times0 + 22\times1 + 12\times2 + 3\times3 + 4\times4 = 71$
Mean = $\Sigma fx \div \Sigma f = 71 \div 50$
Answer: 1.42