Day 42 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Natural logs and e^x (All boards)

Solve 4 ln x = 3.

Give your answer to 3 significant figures.

$\ln x = \frac{3}{4}$

$x = e^{\frac{3}{4}} = 2.12$

Answer: x = 2.12

2 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of seven consecutive integers is a multiple of 7.

Let the integers be n, n + 1, ..., n + 6.

Sum = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) + (n + 6) = 7n + 21

= 7(n + 3), which is a multiple of 7.

Answer: Shown

3 Coding (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The values x are coded using $y = (x - 50) \div 4$.

The coded values have mean $\bar{y} = 3.9$ and standard deviation 1.5.

Find the mean and standard deviation of x.

x = 50 + 4y

Mean: 50 + 4 × 3.9 = 65.6

Standard deviation: 4 × 1.5 = 6 (adding 50 has no effect)

Answer: mean = 65.6, standard deviation = 6

4 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 6 terms of the geometric series 96 + 48 + 24 + ...

$a = 96, r = \frac{1}{2}, n = 6$

$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{96(1 - (\frac{1}{2})^6)}{1 - \frac{1}{2}}$

= 189

Answer: 189

5 Collinear points using vectors (All boards)

A(3, −2), B(15, 6) and C(30, 16).

Show that A, B and C are collinear and find AB : BC.

$\text{AB} = \begin{pmatrix}12\\8\end{pmatrix}$ and $\text{BC} = \begin{pmatrix}15\\10\end{pmatrix}$

$\text{AB} = 4\begin{pmatrix}3\\2\end{pmatrix}$ and $\text{BC} = 5\begin{pmatrix}3\\2\end{pmatrix}$, so AB and BC are parallel with common point B

Hence A, B, C are collinear and AB : BC = 4 : 5

Answer: collinear; AB : BC = 4 : 5

6 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(x^2 + 2)$ for x = 1, 2, 3, 4.

(a) Find the value of k.

(b) Find $P(X \geq 2)$.

(a) Probabilities sum to 1: k(3 + 6 + 11 + 18) = 1

$k = \frac{1}{38}$

(b) $P(X \geq 2) = \frac{6}{38} + \frac{11}{38} + \frac{18}{38}$

Answer: (a) k = $\frac{1}{38}$ (b) $\frac{35}{38}$

7 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 6x^2 + 9x$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 12x + 9$
$dy/dx = 0$ gives $x = 1$ or $x = 3$
Substitute: $y = 4$ at $x = 1$ and $y = 0$ at $x = 3$
Answer: A (1, 4), B (3, 0)

8 Friction: acceleration along rough ground (All boards)

A block of mass 10 kg is pulled along rough horizontal ground by a horizontal force of 69 N.
The coefficient of friction is 0.5. Take $g = 9.8$ m s^{-2}. Find the acceleration.

$R = 10 \times 9.8 = 98$
$F = \mu R = 0.5 \times 98 = 49$
$P - F = ma$: $69 - 49 = 10a$
Answer: 2 m s^{-2}

9 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-5x - 7}{(x - 1)(x + 3)}$ in partial fractions.

Write $\frac{-5x - 7}{(x - 1)(x + 3)} = \frac{A}{(x - 1)} + \frac{B}{(x + 3)}$
$-5x - 7 = A(x + 3) + B(x - 1)$
$x = 1$: $-12 = A(4)$ so $A = -3$
$x = -3$: $8 = B(-4)$ so $B = -2$
Answer: $\frac{-3}{(x - 1)} + \frac{-2}{(x + 3)}$

10 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a boat is $(-6\mathbf{i} - 8\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{6^2 + 8^2} = \sqrt{100}$
(b) $\tan\theta = \frac{8}{6}$
$\theta = \tan^{-1}(\frac{8}{6})$
Answer: (a) 10 m s^{-1} (b) 53.1°