Day 41 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 7 values: 17, 10, 5, 13, 10, 18, 13
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 86$, $\Sigma x^2 = 1176$, $n = 7$
Mean = $86 \div 7 = 12.29$
Variance = $\frac{1176}{7} - (12.29)^2 = 17.06$
Standard deviation = $\sqrt{17.06}$
Answer: mean = 12.3, standard deviation = 4.13

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 4x - 2$ and $g(x) = x^2 + 1$.
Find (a) fg(x), (b) gf(x).

$fg(x) = f(x^2 + 1) = 4x^2 + 2$
$gf(x) = (4x - 2)^2 + 1 = 16x^2 - 16x + 5$
Answer: (a) $4x^2 + 2$ (b) $16x^2 - 16x + 5$

3 Velocity-time: constant deceleration to rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car decelerates uniformly from 24 m s^{-1} to rest in 6 s. Find (a) the deceleration, (b) the distance travelled while decelerating.

(a) $a = \frac{v - u}{t} = \frac{0 - 24}{6}$
(b) $s = \frac{(u + v)}{2} t = 12 \times 6$
Answer: (a) 4 m s^{-2} (b) 72 m

4 Find the angle of a sector (All boards)

A sector has radius 5 cm and area 15.0 cm^2.
Find the angle of the sector in radians and the arc length.

$\frac{1}{2} \times 5^2 \times \theta = 15.0$, so $\theta = 1.2$
Arc length = $r\theta = 5 \times 1.2 = 6.00$ cm
Answer: θ = 1.2 radians; arc length 6.00 cm

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

20 pairs of data on age of a car and its value have $r = 0.45$.
Test, at the 5% significance level, for evidence of positive correlation. The critical value is 0.3783.

$H_0: \rho = 0$, $H_1: \rho > 0$
$r = 0.45$ is beyond the critical value 0.3783
Reject H_0
Answer: Reject H_0: there is evidence of positive correlation

6 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence begins 128, 192, 288, ...
Find the common ratio and the 8th term.

$r = \frac{192}{128} = \frac{3}{2}$
$u_n = ar^{n-1}$, so $u_8 = 128 \times (\frac{3}{2})^7$
$= 2187$
Answer: $r = \frac{3}{2}$, $u_8 = 2187$

7 Rewrite a sin x + b cos x in R form (All boards)

Express 24 sin x + 7 cos x in the form R cos(x − α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

R cos(x − α) = R cos x cos α + R sin x sin α
R cos α = 7 and R sin α = 24
R = $\sqrt{24^2 + 7^2}$ = 25, tan α = $\frac{24}{7}$, α = 73.7°

Answer: 25 cos(x − 73.7°)

8 Rough slope: limiting equilibrium or sliding (All boards)

A particle slides down a rough plane inclined at angle α to the horizontal, where sin α = $\frac{3}{5}$ and the coefficient of friction is 0.25. Take g = 9.8 m s^{-2}. Find the acceleration.

R = mg cos α = m × 9.8 × $\frac{4}{5}$
mg sin α − μR = ma
a = 9.8 × $\frac{3}{5}$ − 0.25 × 9.8 × $\frac{4}{5}$

Answer: 3.92 m s^{-2}

9 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When $x^3 + kx^2 - 3x + 39$ is divided by (x + 3), the remainder is 3. Find the value of k.

f(−3) = 3
−27 + 9k + 48 = 3
9k = −18
Answer: k = −2

10 Maximum value from R form (All boards)

f(x) = 5 sin x + 12 cos x + 3.
Write f(x) in the form R sin(x + α) + 3, with α in degrees to 1 d.p., and state the maximum value of f(x).

R = $\sqrt{5^2 + 12^2}$ = 13, tan α = $\frac{12}{5}$ so α = 67.4°
f(x) = 13 sin(x + 67.4°) + 3
sin(x + α) has maximum value 1, so f(max) = 13 + 3
Answer: f(x) = 13 sin(x + 67.4°) + 3; maximum 16