Day 40 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = 8\mathbf{i} + 6\mathbf{j}$.

Find (a) $|\mathbf{p}|$, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{8^2 + 6^2} = 10$

(b) Acute angle $= \tan^{-1}\left(\frac{6}{8}\right) = 36.9°$

p is in the first quadrant, so the angle is 36.9°

Answer: (a) 10 (b) 36.9°

2 Outliers (Edexcel, OCR, MEI)

For a data set $Q_1 = 58$ and $Q_3 = 86$.

A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.

Determine whether 10 and 15 are outliers.

IQR = 86 − 58 = 28, so 1.5 × IQR = 42

Lower limit = 58 − 42 = 16; upper limit = 86 + 42 = 128

10 is an outlier; 15 is an outlier

Answer: 10 is an outlier; 15 is an outlier

3 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(3\mathbf{i} + 3\mathbf{j})$ N, $(p\mathbf{i} + q\mathbf{j})$ N and $(-2\mathbf{i} - 7\mathbf{j})$ N act on a particle which is in equilibrium. **i** and **j** are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:

i components: $p + (3) + (-2) = 0$

j components: $q + (3) + (-7) = 0$

Answer: $p = -1$, $q = 4$

4 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each shot hits the target with probability 0.5.

Find the probability of at least one hit in 7 shots, to 3 s.f.

P(zero) $= 0.5^7$

P(at least one) $= 1 - 0.5^7$

Answer: 0.992

5 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 3$.

Use the Newton-Raphson method with $x_0 = 1$ to find x_1,

giving your answer to 3 decimal places.

$f'(x) = e^x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 1 - \frac{-0.2817}{2.718} = 1.1036$$

Answer: $x_1 = 1.104$

6 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 12 and common ratio $\frac{1}{4}$.

Find the sum to infinity.

$|r| < 1$, so the series converges.

$$S_\infty = \frac{a}{1 - r} = \frac{12}{1 - \frac{1}{4}}$$

$= 16$

Answer: 16

7 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 3 : 1.
Find AB and OP in terms of $\mathbf{a}$ and $\mathbf{b}$.

AB = OB − OA = $\mathbf{b} - \mathbf{a}$

AP = $\frac{3}{4}$ AB = $\frac{3}{4}(\mathbf{b} - \mathbf{a})$

OP = OA + AP = $\mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a})$

= $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Answer: AB = $\mathbf{b} - \mathbf{a}$; OP = $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

8 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(3x^4)^2 \div 9x^{-2}$.

$(3x^4)^2 = 9x^8$

$9x^8 \div 9x^{-2}$: divide the numbers and subtract the powers

$= x^{10}$

Answer: x^{10}

9 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 7 m s^{-1}.
At that instant car B starts from rest at O with constant acceleration 1 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $7t = \frac{1}{2} \times 1 \times t^2$

$t(7 - 0.5t) = 0$, so $t = 0$ or $t = 7 \div 0.5$

Answer: 14 s

10 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$(x - 2)$ is a factor of $x^3 + kx^2 + 5x - 6$.
Find the value of k.

Let $f(x) = x^3 + kx^2 + 5x - 6$. Then $f(2) = 0$:

$8 + 4k + 4 = 0$

$4k = -12$

Answer: $k = -3$