Day 38 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = (x + 4)(x - 1)(x - 4)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -4, 1, 4$

$x = 0$: $y = 4 \times (-1) \times (-4) = 16$

Answer: $(-4, 0)$, $(1, 0)$, $(4, 0)$, $(0, 16)$

2 Find the angle of a sector (All boards)

A sector has radius 4 cm and arc length 6.40 cm.

Find the angle of the sector in radians and the area of the sector.

$\theta = \frac{\text{arc length}}{r} = \frac{6.40}{4} = 1.6$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 4^2 \times 1.6 = 12.8\ \text{cm}^2$

Answer: $\theta = 1.6$ radians; area $12.8\ \text{cm}^2$

3 Binomial: mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(30, 0.25)$. Find E(X) and Var(X).

$E(X) = np = 30 \times 0.25 = 7.5$

$\text{Var}(X) = npq = 30 \times 0.25 \times 0.75$

Answer: E(X) = 7.5, Var(X) = 5.625

4 Addition formulae: exact values (All boards)

By writing 105° as 45° + 60°, find the exact value of sin 105°.

$\sin(45° + 60°) = \sin 45° \cos 60° + \cos 45° \sin 60°$

$= \frac{\sqrt{2}}{2} \times \frac{1}{2} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}$

Simplify: $\frac{\sqrt{6} + \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} + \sqrt{2}}{4}$

5 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(50, p)$ and $E(X) = 12.5$.

Find p and Var(X).

$np = 12.5$, so $p = 12.5 \div 50 = 0.25$

$\text{Var}(X) = npq = 12.5 \times 0.75$

Answer: p = 0.25, Var(X) = 9.375

6 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 8 terms of the series

$4 + 7 + 10 + 13 + \ldots$

$a = 4$, $d = 3$, $n = 8$

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{8}{2}[2(4) + 7 \times 3]$

$= 116$

Answer: 116

7 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry travelling at 20 m s^{-1} brakes uniformly and comes to rest after travelling 40 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 20^2 + 2 \times a \times 40$
so $a = -5$, a deceleration of 5

(b) $s = \frac{(u + v)}{2} t$: $40 = 10\,t$

Answer: (a) 5 m s^{-2} (b) 4 s

8 Rewrite a sin x + b cos x in R form (All boards)

Express 6 sin x + 8 cos x in the form R cos(x − α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

$R\cos(x - \alpha) = R\cos x \cos\alpha + R\sin x \sin\alpha$
$R\cos\alpha = 8$ and $R\sin\alpha = 6$

$R = \sqrt{6^2 + 8^2} = 10$, $\tan\alpha = \frac{6}{8}$, $\alpha = 36.9°$

Answer: 10 cos(x − 36.9°)

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 4x + k$ is a tangent to the curve
$y = x^2 + 2x - 1$.
Find the value of k.

$x^2 + 2x - 1 = 4x + k$
$x^2 - 2x + (-1 - k) = 0$
A tangent touches once: discriminant = 0
$(-2)^2 - 4(-1 - k) = 0$
Answer: k = −2

10 Vertical motion: greatest height from speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cricket ball is projected vertically upwards with speed 28 m s^{-1} from ground level. Taking g = 9.8 m s^{-2}, find the maximum height it reaches above the ground.

At the top $v = 0$: $v^2 = u^2 + 2as$
$0 = 28^2 - 2 \times 9.8 \times s$
$s = \frac{784}{19.6}$
Answer: 40 m