Day 37 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = t^3 + 4t^2 - 4t + 5$.
Find its velocity when t = 2.

$v = \frac{ds}{dt} = 3t^2 + 8t - 4$

When t = 2: $v = 3 \times 2^2 + 8 \times 2 - 4$

Answer: 24 m s^{-1}

2 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In an arithmetic sequence $u_3 = 11$ and $u_7 = 19$.
Find the first term and the common difference.

Subtract: $(7 - 3)d = 19 - 11$
$4d = 8$, so $d = 2$
$a = u_3 - 2d = 11 - 4 = 7$

Answer: a = 7, d = 2

3 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = \frac{3}{x_n^2 + 1} + 1$, with $x_0 = 2$.

Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 1.6000$
$x_2 = 1.8427$
$x_3 = 1.6825$

Answer: $x_1 = 1.600$, $x_2 = 1.843$, $x_3 = 1.683$

4 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^3 - 4x^2 + 4x - 3$.
C is reflected in the y-axis. Find the equation of the image.

Reflection in the y-axis: replace x by $-x$.

$y = -x^3 - 4x^2 - 4x - 3$

Answer: $y = -x^3 - 4x^2 - 4x - 3$

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

12 pairs of data on daily mean temperature and sunshine hours have r = 0.51.
Test, at the 5% significance level, for evidence of correlation. The critical value is ±0.5760.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
r = 0.51 is not beyond the critical value ±0.5760
Do not reject H_0

Answer: Do not reject H_0: there is not enough evidence of correlation

6 Rewrite a sin x + b cos x in R form (All boards)

Express 5 sin x + 12 cos x in the form R cos(x − α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

$R\cos(x - \alpha) = R\cos x \cos\alpha + R\sin x \sin\alpha$
$R\cos\alpha = 12$ and $R\sin\alpha = 5$

$R = \sqrt{5^2 + 12^2} = 13$, $\tan\alpha = \frac{5}{12}$, $\alpha = 22.6°$

Answer: 13 cos(x − 22.6°)

7 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{3}{8}x^2$ for $0 \leq x \leq 2$, and 0 otherwise.

Find $P(X < 1)$.

$P(X < 1) = \int$ from 0 to 1 of $\frac{3}{8}x^2\,dx$

$= [\frac{3}{24}x^3]$ from 0 to 1

Answer: $\mathbf{\frac{1}{8}}$

8 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 3 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}.

Find (a) its acceleration, (b) its speed after 5 s.

(a) Parallel to slope: $mg\sin\alpha = ma$

$a = 9.8 \times \frac{3}{5}$

(b) $v = u + at = 0 + 5.88 \times 5$

Answer: (a) 5.88 m s^{-2} (b) 29.4 m s^{-1}

9 Solve trig equation (calculator, degrees) (All boards)

Solve $\sin x = 0.45$ for $0° \leq x \leq 360°$.

Give your answers to 1 decimal place.

Reference angle: $\sin^{-1}(0.450) = 26.7°$

sin is positive in the first and second quadrants.

$x = 26.7°$ or $x = 153.3°$

Answer: x = 26.7°, 153.3°

10 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 2x)^{-3}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -3$, $y = 2x$

$= 1 + (-3)(2x) + \frac{(-3)((-4))}{2}(2x)^2$

$= 1 - 6x + 24x^2$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 - 6x + 24x^2$; $|x| < \frac{1}{2}$