Day 365 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Degrees and radians (All boards)

(a) Convert 120° to radians, giving your answer in terms of π.

(b) Convert $\frac{3\pi}{5}$ radians to degrees.

(a) $120° = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$

(b) $\frac{3\pi}{5}$ radians $= \frac{3\pi}{5} \times \frac{180}{\pi} = 108°$

Answer: (a) $\frac{2\pi}{3}$ radians (b) 108°

2 Regression: interpreting coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

A model relates the amount, y litres, left in a tank to the time, t hours, since it was filled by $y = 77 - 7.1t$.
Interpret the value 7.1 in this context.

The gradient is the change in y for each 1 unit of t.
For each extra hour, y decreases by 7.1 litres.
Answer: each extra hour, y decreases by 7.1 litres

3 3D vectors: distance, magnitude and midpoint (All boards)

A has coordinates (−3, −1, 2) and B has coordinates (−9, 8, 0).
Find (a) the vector AB, (b) the distance AB, (c) the midpoint of AB.

(a) $\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} -6 \\ 9 \\ -2 \end{pmatrix}$

(b) $|\text{AB}| = \sqrt{6^2 + 9^2 + 2^2} = 11$

(c) Midpoint $= (-6, \frac{7}{2}, 1)$

Answer: (a) $\begin{pmatrix} -6 \\ 9 \\ -2 \end{pmatrix}$ (b) 11 (c) $(-6, \frac{7}{2}, 1)$

4 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = 5t$, $y = \frac{6}{t}$, $t \neq 0$.

Find a Cartesian equation of the curve.

$x = 5t$ gives $t = \frac{x}{5}$

$y = \frac{6}{t} = 6 \div \frac{x}{5} = \frac{30}{x}$

Answer: $y = \frac{30}{x}$

5 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 8$) is $v = (-2t^2 + 16t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 16 - 4t$

Maximum when $a = 0$: $t = \frac{16}{4} = 4$

$v = 16 \times 4 - 2 \times 4^2$

Answer: 32 m s^{-1} at t = 4

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6 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The nth term of an arithmetic sequence is 2n + 3.
Find the smallest n for which the term exceeds 400.

$2n + 3 > 400$

$n > \frac{397}{2} = 198.50...$

The smallest integer n is 199.

Answer: n = 199

7 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points P and Q, 252 m apart, and cycle directly towards each other at constant speeds of 11 m s^{-1} and 7 m s^{-1}. Find the time until they meet.

Closing speed = 11 + 7 = 18 m s^{-1}

Time = $\frac{252}{18}$

Answer: 14 s

8 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{1}{8}x$ for $0 \le x \le 4$, and 0 otherwise.

Find P(X < 1).

$P(X < 1) = \int$ from 0 to 1 of $\frac{1}{8}x\,dx$

$= [\frac{1}{16}x^2]$ from 0 to 1

Answer: $\frac{1}{16}$

9 Optimisation (algebraic set-up) (All boards)

A farmer has 20 m of fencing for three sides of a rectangular pen; the fourth side is a wall.
Find the maximum area of the pen.

Two sides of length x m: $A = x(20 - 2x) = 20x - 2x^2$
$dA/dx = 20 - 4x = 0$, so $x = 5$
$d^2A/dx^2 = -4 < 0$ (maximum); $A = 5 \times 10$
Answer: 50 m^2

10 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 6x + 4 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x + 3)^2 - 9 + 4 = 0$
$(x + 3)^2 = 5$
$x + 3 = \pm\sqrt{5}$
Answer: $x = -3 \pm \sqrt{5}$