Day 363 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

M(5, 10) is the midpoint of the line segment AB.
A is the point (2, 5). Find the coordinates of B.

x-coordinate: $\frac{2 + x_B}{2} = 5$, so $x_B = 8$

y-coordinate: $\frac{5 + y_B}{2} = 10$, so $y_B = 15$

Answer: B(8, 15)

2 Solve a^x = b using logarithms (All boards)

Solve $5^{x+3} = 200$.
Give your answer to 3 significant figures.

Take logs: $(x + 3) \log 5 = \log 200$
$x + 3 = 3.292$, so $x = 0.292$
Answer: x = 0.292

3 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-x^3 - 2x^2 - 7x + 1$ is divided by $(x + 2)$.

Remainder = $f(-2)$
$= 8 + (-8) + 14 + 1$
$= 15$
Answer: 15

4 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a building of height 78.4 m. Take g = 9.8 m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $78.4 = \frac{1}{2} \times 9.8 \times t^2$

$78.4 = 4.9t^2$, so $t^2 = 16$
(b) $v = u + at = 9.8 \times 4$
Answer: (a) 4 s (b) 39.2 m s^{-1}

5 Collinear points using vectors (All boards)

A(3, 2), B(6, −1) and C(11, −6).
Show that A, B and C are collinear and find AB : BC.

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$

$\overrightarrow{AB} = 3\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\overrightarrow{BC} = 5\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, so AB and BC are parallel with common point B
Hence A, B, C are collinear and AB : BC = 3 : 5
Answer: collinear; AB : BC = 3 : 5

6 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 5 values: 19, 11, 9, 14, 2
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 55$, $\Sigma x^2 = 763$, $n = 5$
Mean = $55 \div 5 = 11.00$

Variance = $\frac{763}{5} - (11.00)^2 = 31.60$

Standard deviation = $\sqrt{31.60}$
Answer: mean = 11.0, standard deviation = 5.62

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7 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x + k$ is a tangent to the curve $y = x^2 - 4x$.
Find the value of k.

$x^2 - 4x = -2x + k$
$x^2 - 2x + (0 - k) = 0$
A tangent touches once: discriminant = 0
$(-2)^2 - 4(0 - k) = 0$
Answer: $k = -1$

8 Integrate using partial fractions (All boards)

$\frac{6x + 16}{(x + 2)(x + 3)} \equiv \frac{A}{x + 2} + \frac{B}{x + 3}$.
Find A and B, and hence find $\int \frac{6x + 16}{(x + 2)(x + 3)}\, dx$.

$6x + 16 \equiv A(x + 3) + B(x + 2)$
$x = -2$ gives $A = 4$; $x = -3$ gives $B = 2$
$\int \left(\frac{4}{x + 2} + \frac{2}{x + 3}\right) dx = 4 \ln|x + 2| + 2 \ln|x + 3| + c$
Answer: A = 4, B = 2; 4 ln |x + 2| + 2 ln |x + 3| + c

9 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces (0) N and $(8\mathbf{i} + 4\mathbf{j})$ N act on a particle of mass 4 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(0) + (8\mathbf{i} + 4\mathbf{j})$
$= (8\mathbf{i} + 4\mathbf{j})$ N
F = ma: $a = \frac{1}{4}(8\mathbf{i} + 4\mathbf{j})$
Answer: $(2\mathbf{i} + \mathbf{j})$ m s^{-2}

10 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(10, p)$. H_0: $p = 0.1$, H_1: $p > 0.1$.
In a sample, $x = 3$. It is given that $P(X \geq 3) = 0.0702$.
Test H_0 at the 10% significance level.

Compare 0.0702 with 0.1
$0.0702 < 0.1$, so reject H_0
Answer: Reject H_0: evidence that p is greater than 0.1