Day 362 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 7 terms of the geometric series 3 + 9 + 27 + ...

$a = 3$, $r = 3$, $n = 7$

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{3(3^7 - 1)}{3 - 1}$

$= 3279$

Answer: 3279

2 Velocity-time: constant deceleration to rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car decelerates uniformly from 12 m s^{-1} to rest in 8 s. Find (a) the deceleration, (b) the distance travelled while decelerating.

(a) $a = \frac{v - u}{t} = \frac{0 - 12}{8}$

(b) $s = \frac{(u + v)}{2} t = 6 \times 8$

Answer: (a) 1.5 m s^{-2} (b) 48 m

3 Evaluating fractional and negative indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $32^{\frac{2}{5}}$.

$32^{\frac{2}{5}} = (32^{\frac{1}{5}})^2$

$32^{\frac{1}{5}} = 2$

$(2)^2 = 4$

Answer: 4

4 3D vectors: distance, magnitude and midpoint (All boards)

A has coordinates (1, 5, 3) and B has coordinates (7, 7, 0).
Find (a) the vector AB, (b) the distance AB, (c) the midpoint of AB.

(a) $AB = OB - OA = \begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix}$

(b) $|AB| = \sqrt{6^2 + 2^2 + 3^2} = 7$

(c) Midpoint = $(4, 6, \frac{3}{2})$

Answer: (a) $\begin{pmatrix} \mathbf{6} \\ \mathbf{2} \\ \mathbf{-3} \end{pmatrix}$ **(b) 7 (c)** $\mathbf{(4, 6, \frac{3}{2})}$

5 Combined mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mean of 10 times (seconds) of the Class A is 45. The mean of 30 times (seconds) of the Class B is 55.
Find the mean of all 40 values.

Total of first group = 10 × 45 = 450
Total of second group = 30 × 55 = 1650
Mean = 2100 ÷ 40
Answer: 52.5

6 Car towing a caravan (All boards)

A car of mass 1200 kg tows a caravan of mass 500 kg on a level road with acceleration 1 m s^{-2}. Resistances: 200 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 200 - 100 = 1700 \times 1$
Caravan: $T - 100 = 500 \times 1$
Answer: D = 2000 N, T = 600 N

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7 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Use proof by exhaustion to show that $n^2 + n + 4$ is divisible by 2 for every integer n with $1 \leq n \leq 4$.

Substitute each value of n:
n = 1: 6 = 2 × 3 ; n = 2: 10 = 2 × 5
n = 3: 16 = 2 × 8 ; n = 4: 24 = 2 × 12
Every value is a multiple of 2, so the statement is true for $1 \leq n \leq 4$.
Answer: Shown

8 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = e^{\frac{x}{2}}$ from x = 0 to x = 4 divided into 4 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 1; y values: 1.000, 1.649, 2.718, 4.482, 7.389

Area $\approx \frac{1}{2}[1.000 + 7.389 + 2(8.849)]$

= 13.0
Answer: 13.0

9 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(15, p). H_0: p = 0.3, H_1: p < 0.3.
Find the critical region for a test at the 10% significance level.

P(X ≤ 1) = 0.0353 ≤ 0.1
P(X ≤ 2) = 0.127 > 0.1
Answer: $X \leq 1$

10 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $3x^2 + 3y^2 = 15$.
Find dy/dx in terms of x and y, and hence the gradient at (2, 1).

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = -\frac{3x}{3y}$

At (2, 1): $dy/dx = -2$

Answer: $dy/dx = -\frac{3x}{3y}$; gradient -2