Day 360 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity $(\mathbf{i})$ m s^{-1} and constant acceleration $(2\mathbf{i} - \mathbf{j})$ m s^{-2}.
Find its velocity after 2 s.

$v = u + at$
$v = (\mathbf{i}) + 2(2\mathbf{i} - \mathbf{j})$
$= (\mathbf{i}) + (4\mathbf{i} - 2\mathbf{j})$
Answer: $(5\mathbf{i} - 2\mathbf{j})$ m s^{-1}

2 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{x + 6} - 4$.
State the largest possible domain and the range of f.

$x + 6 \geq 0$, so $x \geq -6$.
$\sqrt{\ldots} \geq 0$, so $f(x) \geq -4$.
Answer: Domain $x \geq -6$, range $f(x) \geq -4$

3 Trapezium rule from a table (AQA, Edexcel, OCR, MEI, Eduqas)

The table shows values of y = f(x).
x: 1 1.5 2 2.5 3
y: 5.9 3.6 8.2 4.2 4.7
Use the trapezium rule to estimate the area under the curve between x = 1 and x = 3.

h = 0.5 with 4 strips
Area $\approx \frac{0.5}{2}[5.9 + 4.7 + 2(3.6 + 8.2 + 4.2)]$
$= \frac{0.5}{2} \times 42.6$
$= 10.65$
Answer: 10.65

4 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the first four terms, in ascending powers of x, of the expansion of $(1 + 3x)^6$.

$(1 + y)^6 = 1 + 6y + 15y^2 + 20y^3 + \ldots$ with $y = 3x$
$= 1 + 6(3x) + 15(3x)^2 + 20(3x)^3$
Answer: $1 + 18x + 135x^2 + 540x^3$

5 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 2 sin 2x
(b) e^{2x} + sin 2x

(a) $dy/dx = 4 \cos 2x$
(b) $dy/dx = 2e^{2x} + 2 \cos 2x$
Answer: (a) $4 \cos 2x$ (b) $2e^{2x} + 2 \cos 2x$

6 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 8^2)$. H_0: $\mu = 35$, H_1: $\mu > 35$.
A sample of 64 has mean $\bar{x} = 37.5$.
Test at the 5% level. Critical value: z = 1.645.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (37.5 - 35) \div (8 \div 8)$
$z = 2.50$
2.50 is inside the critical region, so reject H_0
Answer: $z = 2.50$; reject H_0

Day 360 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Optimisation (algebraic set-up) (All boards)

A rectangle has perimeter 60 cm.
Find the maximum possible area of the rectangle and the dimensions that give it.

Sides x and 30 − x, so $A = x(30 - x) = 30x - x^2$
$dA/dx = 30 - 2x = 0$, so $x = 15$
$d^2A/dx^2 = -2 < 0$, so it is a maximum; $A = 15 \times 15$
Answer: A = 225 cm², a 15 cm square

8 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^3 + 9x^2 + 14x - 20$ by $(x + 4)$.
Find the quotient and the remainder.

Use synthetic division with $a = -4$:
Bottom row: 1, 5, −6, 4
Quotient: $x^2 + 5x - 6$
Remainder: 4
Answer: Quotient $x^2 + 5x - 6$, remainder 4

9 Power of an engine: driving force (CIE)

A train's engine works at a constant rate of 16 kW. Find the driving force when the vehicle is moving at 20 m s^{-1}.

$P = Fv$
$16000 = F \times 20$
Answer: 800 N

10 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 20 and 40 minutes.

Frequency = frequency density × class width
$20 \leq t < 35$: $2 \times 15 = 30$
$35 \leq t < 40$: $4 \times 5 = 20$
Total = 30 + 20
Answer: 50