Day 36 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Collinear points using vectors (All boards)

A(−1, 0), B(7, −4) and C(13, k).
Given that A, B and C are collinear, find k.

$\text{AB} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$, $\text{AC} = \begin{pmatrix} 14 \\ k - 0 \end{pmatrix}$

AC = $\frac{7}{4}$ AB, so 14 = $\frac{7}{4} \times 8$, giving the multiple $\frac{7}{4}$

$k - 0 = \frac{7}{4} \times (-4)$, so $k = -7$

Answer: $k = -7$

2 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 7t + 12)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 7t + 12 = 0$
$(t - 3)(t - 4) = 0$

(b) $a = \frac{dv}{dt} = 2t - 7$; at $t = 3$, $a = 6 - 7$

Answer: (a) $t = 3$ and $t = 4$ (b) -1 m s^{-2}

3 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence begins 4, −8, 16, ...
Find the common ratio and the 7th term.

$r = \frac{-8}{4} = -2$

$u_n = ar^{n-1}$, so $u_7 = 4 \times (-2)^6$
$= 256$
Answer: $r = -2$, $u_7 = 256$

4 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 3) and (3, $\frac{3}{8}$).

Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 3$

$x = 3$: $3 \times b^3 = \frac{3}{8}$, so $b^3 = \frac{1}{8}$

$b = \frac{1}{2}$

Answer: $a = 3$, $b = \frac{1}{2}$

5 Binomial: P(X = k) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(12, 0.5). Find P(X = 6), giving your answer to 3 s.f.

$P(X = 6) = {}^{12}C_6 \times 0.5^6 \times 0.5^6$

$= 924 \times 0.5^6 \times 0.5^6$
Answer: 0.226

6 Resolving a force into components (All boards)

A rope pulls a box with a force of 10 N at an angle of 60° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 10 cos 60°
Vertical = 10 sin 60°
Answer: horizontal 5.00 N, vertical 8.66 N

7 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x + 1)^2 + (y + 1)^2 = 29$ passes through $P(-3, -6)$.
Find the gradient of the tangent to the circle at P.

Centre $C(-1, -1)$.

Gradient of CP = $\frac{-6 - (-1)}{-3 - (-1)} = \frac{5}{2}$

The tangent is perpendicular to CP: gradient = $-1 \div \frac{5}{2}$

Answer: $\mathbf{-\frac{2}{5}}$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.3. If so, the probability that Sam is late is 0.7; otherwise it is 0.4.
Given that Sam is late, find the probability that it rains.

P(both) = 0.3 × 0.7 = 0.21; P(not first and second) = 0.7 × 0.4 = 0.28
P(second) = 0.21 + 0.28 = 0.49
P = 0.21 ÷ 0.49

Answer: $\mathbf{\frac{3}{7}}$ **= 0.429**

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 3x + k$ is a tangent to the curve
$y = x^2 + x - 2$.
Find the value of k.

$x^2 + x - 2 = 3x + k$
$x^2 - 2x + (-2 - k) = 0$
A tangent touches once: discriminant = 0
$(-2)^2 - 4(-2 - k) = 0$
Answer: k = −3

10 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = 2x(8 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where x = 0 and x = 8

Area = $\int_0^8 2(8x - x^2)\,dx = [2(\frac{8x^2}{2} - \frac{x^3}{3})]$ from 0 to 8

$= 2(\frac{8^3}{2} - \frac{8^3}{3})$

$= \frac{512}{3}$

Answer: $\mathbf{\frac{512}{3}}$