Day 357 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins $-1, 6, 13, 20, \ldots$
Find (a) an expression for the nth term,
(b) the 32th term.

$a = -1$, $d = 7$
$u_n = a + (n - 1)d = -1 + (n - 1)(7) = 7n - 8$
$u_{32} = 216$
Answer: (a) $7n - 8$ (b) 216

2 Regression: interpreting coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

A model relates the mass, m kg, of a puppy to its age, w weeks by $m = 1 + 1.1w$.
Interpret the value 1 in this context.

The constant is the predicted value of m when $w = 0$.
So the predicted m at $w = 0$ is 1 kg.
Answer: predicted m is 1 kg when $w = 0$

3 Driving force and resistance (All boards)

A car of mass 1500 kg is driven along a straight horizontal road by a driving force of 1525 N against a constant resistance of 400 N.
Find the acceleration of the car.

Resultant force $= 1525 - 400 = 1125$ N
$F = ma$: $1125 = 1500\,a$
Answer: 0.75 m s^{-2}

4 Compound growth: find n (All boards)

£250 invested at 4% compound interest per year.
After how many whole years will it first exceed £750?

$250 \times 1.04^n > 750$, so $1.04^n > 3$
$n > \dfrac{\log 3}{\log 1.04} = 28.01$
The least whole number of years is 29
Answer: 29 years

5 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$P(A) = 0.1$, $P(B) = 0.7$ and $P(A \cap B) = 0.01$.
(a) Find $P(A \mid B)$.
(b) Are A and B independent? Give a reason.

(a) $P(A \mid B) = P(A \cap B) \div P(B) = 0.01 \div 0.7 = 0.0142857143$
(b) $P(A) \times P(B) = 0.1 \times 0.7 = 0.07$
This does not equal $P(A \cap B) = 0.01$, so A and B are not independent
Answer: (a) 0.0142857143 (b) not independent, as $P(A) \times P(B) \neq P(A \cap B)$

6 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a ball is $(-5\mathbf{i} + 12\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{5^2 + 12^2} = \sqrt{169}$
(b) $\tan\theta = \dfrac{12}{5}$
$\theta = \tan^{-1}\left(\dfrac{12}{5}\right)$
Answer: (a) 13 m s^{-1} (b) 67.4°

Day 357 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $3\,e^x + 2\ln x$

(b) $3\ln x$

(a) $dy/dx = 3\,e^x + \frac{2}{x}$

(b) $dy/dx = \frac{3}{x}$

Answer: (a) $3\,e^x + \frac{2}{x}$ (b) $\frac{3}{x}$

8 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 6 terms of the geometric series 5 + 15 + 45 + ...

$a = 5$, $r = 3$, $n = 6$

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{5(3^6 - 1)}{3 - 1}$

$= 1820$

Answer: 1820

9 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = (x^2 + 4)^4$
at the point where $x = 1$.

Use the chain rule: $dy/dx = 4(x^2 + 4)^3 \times 2x$

At $x = 1$: $dy/dx = 1000$

Answer: 1000

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve
$y = x^2 + 1$.
Find the value of k.

$x^2 + 1 = -4x + k$

$x^2 + 4x + (1 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(1 - k) = 0$

Answer: $k = -3$