Day 356 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplify using identities (All boards)

Simplify $3 - 3\sin^2 x$.

Take out 3: $3(1 - \sin^2 x) = 3\cos^2 x$

Answer: $3\cos^2 x$

2 Mutually exclusive events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B are mutually exclusive, with P(A) = 0.2 and P(B) = 0.55.
(a) Find P(A ∪ B).
(b) Find the probability that neither A nor B happens.

(a) Mutually exclusive, so $P(A \cap B) = 0$
$P(A \cup B) = 0.2 + 0.55 = 0.75$
(b) $1 - 0.75 = 0.25$
Answer: (a) 0.75 (b) 0.25

3 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 28 terms of the series
6 + 10 + 14 + 18 + ...

$a = 6$, $d = 4$, $n = 28$

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{28}{2}[2(6) + 27 \times 4]$

$= 1680$

Answer: 1680

4 Work done and kinetic energy (CIE)

A particle of mass 2 kg changes its speed from 4 m s^{-1} to 9 m s^{-1}. Find the increase in its kinetic energy.

$KE = \frac{1}{2}mv^2$

Increase $= \frac{1}{2} \times 2 \times 9^2 - \frac{1}{2} \times 2 \times 4^2$

Answer: 65 J

5 Partial fractions: find the constants (All boards)

Express $\frac{3x + 2}{(x + 1)(x + 2)}$ in the form $\frac{A}{x + 1} + \frac{B}{x + 2}$.
Find the values of A and B.

$3x + 2 \equiv A(x + 2) + B(x + 1)$
$x = -1$: $-1 = A(1)$, so $A = -1$
$x = -2$: $-4 = B(-1)$, so $B = 4$
Answer: $A = -1$, $B = 4$

6 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^4 + 3x^3 + x^2 - 11x - 1$ by $(x - 1)$.
Find the quotient and the remainder.

Use synthetic division with a = 1:
Bottom row: 1, 4, 5, −6, −7
Quotient: $x^3 + 4x^2 + 5x - 6$
Remainder: −7
Answer: Quotient $x^3 + 4x^2 + 5x - 6$, remainder −7

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7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 5^2)$. H_0: $\mu = 110$, H_1: $\mu \neq 110$.

A sample of 9 has mean $\bar{x} = 114.33$.
Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (114.33 - 110) \div (5 \div 3)$
$z = 2.60$
2.60 is inside the critical region, so reject H_0
Answer: $z = 2.60$; reject H_0

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve
$y = x^2 - 8x - 2$.
Find the value of k.

$x^2 - 8x - 2 = -4x + k$
$x^2 - 4x + (-2 - k) = 0$
A tangent touches once: discriminant = 0
$(-4)^2 - 4(-2 - k) = 0$
Answer: $k = -6$

9 Trapezium rule from a table (AQA, Edexcel, OCR, MEI, Eduqas)

The table shows values of $y = f(x)$.
x: 0 0.5 1 1.5 2
y: 1.7 3.5 4.9 5.2 3.8
Use the trapezium rule to estimate the area under the curve between $x = 0$ and $x = 2$.

$h = 0.5$ with 4 strips
$\text{Area} \approx \frac{0.5}{2}[1.7 + 3.8 + 2(3.5 + 4.9 + 5.2)]$
$= \frac{0.5}{2} \times 32.7$
$= 8.175$
Answer: 8.175

10 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle starts from rest and moves in a straight line with constant acceleration 1.5 m s^{-2}.
It travels 48 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$
$48 = \frac{1}{2} \times 1.5 \times t^2$
$t^2 = 64$
Answer: 8 s