Day 354 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $7 \leq 4x - 5 < 31$.

Subtract -5 from each part: $12 \leq 4x < 36$
Divide each part by 4.
Answer: $3 \leq x < 9$

2 Reciprocal trig functions: exact values (All boards)

Write down the exact values of
(a) cot 225° (b) cot 120°

(a) $\cot 225° = \frac{1}{\tan 225°} = \frac{1}{1} = 1$

(b) $\cot 120° = \frac{1}{\tan 120°} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

Answer: (a) 1 (b) $-\frac{\sqrt{3}}{3}$

3 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 3 red and 7 blue counters. Two are taken at random without replacement.
Find the probability that at least one of them is red.

$P(\text{no red}) = P(BB) = \frac{7}{10} \times \frac{6}{9} = \frac{7}{15}$

$P(\text{at least one red}) = 1 - \frac{7}{15}$

Answer: $\frac{8}{15}$

4 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, where $f(x)$ is a cubic. Find $f(x)$ in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -2$, 3 and 4.
$f(x) = k(x + 2)(x - 3)(x - 4)$
At (0, 72): $k \times 2 \times (-3) \times (-4) = 72$
$k = 3$
Answer: $f(x) = 3(x + 2)(x - 3)(x - 4)$

5 Exponential growth and decay model (All boards)

A radioactive sample has mass $M = 200\, e^{-0.1t}$ grams after t days.
Find the time for the mass to halve, to 3 significant figures.

$\frac{1}{2} = e^{-0.1t}$

$-0.1t = \ln \frac{1}{2} = -\ln 2$

$t = \frac{\ln 2}{0.1} = 6.93$ days

Answer: $t = 6.93$ days

6 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $x^2 + 12x + 7$ in the form $(x + p)^2 + q$, and state the coordinates of the minimum point of $y = x^2 + 12x + 7$.

$x^2 + 12x = (x + 6)^2 - 36$
$x^2 + 12x + 7 = (x + 6)^2 - 29$
The minimum is where $(x + 6)^2 = 0$, i.e. $x = -6$
Answer: $(x + 6)^2 - 29$; minimum $(-6, -29)$

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7 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = \ln x + 1$ from $x = 1$ to $x = 6$ divided into 4 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

$h = 1.25$; y values: 1.000, 1.811, 2.253, 2.558, 2.792

Area $\approx \frac{1.25}{2}[1.000 + 2.792 + 2(6.622)]$

$= 10.6$

Answer: 10.6

8 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a car.
(a) Find the acceleration in the first 4 s.
(b) Find the deceleration in the last 2 s.
(c) Find the total distance travelled.

(a) $a = \frac{11 - 5}{4}$

(b) $a = \frac{11 - 0}{2}$, a deceleration of 5.5

(c) area $= \frac{1}{2}(5 + 11) \times 4 + 11 \times 12 + \frac{1}{2} \times 2 \times 11$

Answer: (a) 1.5 m s^{-2} (b) 5.5 m s^{-2} (c) 175 m

9 Normal approximation to the binomial (CIE)

$X \sim B(120, 0.6)$. Use a normal approximation to find $P(X \leq 69)$, to 3 s.f.

$np = 72$ and $npq = 28.8$, so use $Y \sim N(72, 28.8)$
Continuity correction: $P(Y < 69.5)$

$z = (69.5 - 72) \div \sqrt{28.8}$

Answer: 0.321

10 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 16 m s^{-1} brakes uniformly and comes to rest after travelling 40 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 16^2 + 2 \times a \times 40$
so $a = -3.2$, a deceleration of 3.2

(b) $s = \frac{(u + v)}{2}$ t: $40 = 8$ t

Answer: (a) 3.2 m s^{-2} (b) 5 s