Day 353 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Constant acceleration: distance from average speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train moving in a straight line with constant acceleration increases its speed from 6 m s^{-1} to 18 m s^{-1} in 5 s.
Find the distance it travels in this time.

$s = \frac{(u + v)}{2} t$

$s = \frac{(6 + 18)}{2} \times 5$

Answer: 60 m

2 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins 1, 4, 7, 10, ...
Find (a) an expression for the nth term,
(b) the 24th term.

$a = 1, d = 3$

$u_n = a + (n - 1)d = 1 + (n - 1)(3) = 3n - 2$

$u_{24} = 70$

Answer: (a) $3n - 2$ (b) 70

3 Change of sign to locate a root (All boards)

$f(x) = x^3 - 3x - 4$.
Show that f(x) = 0 has a root between x = 2 and x = 2.5.

$f(2) = -2.00$

$f(2.5) = 4.12$

f is continuous and there is a change of sign, so a root lies in the interval

Answer: f(2) = −2.00, f(2.5) = 4.12: change of sign

4 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 10 and 20 minutes.

Frequency = frequency density × class width

$10 \leq t < 20$: $3 \times 10 = 30$

Answer: 30

5 Integration by parts (All boards)

Use integration by parts to find

$\int x e^{3x} dx$.

$u = x$, $dv/dx = e^{3x}$; $du/dx = 1$, $v = e^{3x}/3$

$\int u (dv/dx) dx = uv - \int v (du/dx) dx$

Evaluate the remaining integral and simplify

Answer: $\frac{e^{3x}(3x - 1)}{9} + c$

6 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at x = −3, −1 and 4.

$f(x) = k(x + 3)(x + 1)(x - 4)$

At (0, −36): $k \times 3 \times 1 \times (-4) = -36$

$k = 3$

Answer: f(x) = 3(x + 3)(x + 1)(x − 4)

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7 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = (2x_n + 10)^{\frac{1}{3}}$, with $x_0 = 1$.
Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.2894$
$x_2 = 2.4429$
$x_3 = 2.4599$
Answer: $x_1 = 2.289$, $x_2 = 2.443$, $x_3 = 2.460$

8 Resolving a force into components (All boards)

A rope pulls a box with a force of 30 N at an angle of 45° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 30 cos 45°
Vertical = 30 sin 45°
Answer: horizontal 21.2 N, vertical 21.2 N

9 Normal approximation to the binomial (CIE)

X ~ B(150, 0.2). Use a normal approximation to find P(X ≤ 35), to 3 s.f.

np = 30 and npq = 24, so use Y ~ N(30, 24)
Continuity correction: P(Y < 35.5)
$z = (35.5 - 30) \div \sqrt{24}$
Answer: 0.869

10 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = -1$ and $y = -1$, so $a = -1$ and $c = -1$.

Through (0, 2): $2 = \frac{k}{0 - (-1)} + -1$

$\frac{k}{1} = 3$, so $k = 3$

Answer: $a = -1$, $c = -1$, $k = 3$