Day 352 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a particle is $(-4\mathbf{i} + 3\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{4^2 + 3^2} = \sqrt{25}$

(b) $\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(\frac{3}{4})$

Answer: (a) 5 m s^{-1} (b) 36.9°

2 Types of data (AQA, Edexcel, OCR, MEI, Eduqas)

Classify each variable as qualitative, discrete or continuous.
(a) the number of texts sent in a day
(b) the make of a car
(c) the volume of water in a bottle

(a) discrete: it is counted, so takes separate values
(b) qualitative: it is described by words, not numbers
(c) continuous: it is measured, so can take any value in a range
Answer: (a) discrete; (b) qualitative; (c) continuous

3 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.
Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length $= r\theta = 6 \times \frac{5\pi}{6} = 5\pi$ cm

(b) Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times \frac{5\pi}{6} = 15\pi$ cm^2

Answer: (a) 5π cm (b) 15π cm^2

4 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{36x^4} \times 5x^2$.

$\sqrt{36x^4} = (36x^4)^{\frac{1}{2}} = 6x^2$
$6x^2 \times 5x^2 = 30x^4$
Answer: $30x^4$

5 Integration by parts (All boards)

Use integration by parts to find
$\int x \cos 4x \, dx$.

$u = x$, $dv/dx = \cos 4x$; $du/dx = 1$, $v = \frac{\sin 4x}{4}$
$\int u \, (dv/dx) \, dx = uv - \int v \, (du/dx) \, dx$
Evaluate the remaining integral and simplify
Answer: $\frac{x \sin 4x}{4} + \frac{\cos 4x}{16} + c$

6 Energy: falling object (potential and kinetic energy) (CIE)

A particle of mass 10 kg is released from rest and falls 44.1 m. Take g = 9.8 m s^{-2}. Use energy to find (a) the loss in potential energy (3 s.f.), (b) the speed of the particle after falling 44.1 m.

(a) Loss in PE $= mgh = 10 \times 9.8 \times 44.1 = 4321.8$

(b) $\frac{1}{2} \times 10 \times v^2 = 4321.8$

$v^2 = 864.36$

Answer: (a) 4320 J (b) 29.4 m s^{-1}

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7 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both

$2x - 2 > 4$ and $x^2 - 7x < 0$.

$2x - 2 > 4$ gives $x > 3$.

$x^2 - 7x < 0$: $(x + 0)(x - 7) < 0$, so $0 < x < 7$.

Both are true where the two ranges overlap.

Answer: $3 < x < 7$

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.6$, H_1: $p > 0.6$.

Find the critical region for a test at the 1% significance level.

$P(X \geq 18) = 0.00361 \leq 0.01$

$P(X \geq 17) = 0.0160 > 0.01$

Answer: $X \geq 18$

9 Maximum value from R form (All boards)

$f(x) = 21 \sin x + 20 \cos x + 3$.

Write f(x) in the form $R \sin(x + \alpha) + 3$, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{21^2 + 20^2} = 29$, $\tan \alpha = \frac{20}{21}$ so $\alpha = 43.6°$

$f(x) = 29 \sin(x + 43.6°) + 3$

$\sin(x + \alpha)$ has maximum value 1, so f(max) = 29 + 3

Answer: $f(x) = 29 \sin(x + 43.6°) + 3$; maximum 32

10 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + 8x^2 + 19x + 12$

(a) Show that $(x + 4)$ is a factor of f(x).

(b) Factorise f(x) completely.

$f(-4) = (-64) + 128 + (-76) + 12 = 0$, so $(x + 4)$ is a factor.

$f(x) = (x + 4)(x^2 + 4x + 3)$

$x^2 + 4x + 3 = (x + 3)(x + 1)$

Answer: (b) $f(x) = (x + 4)(x + 3)(x + 1)$