Day 35 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 5) and (3, 625).
Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 5$
$x = 3$: $5 \times b^3 = 625$, so $b^3 = 125$
$b = 5$
Answer: a = 5, b = 5

2 Median and interquartile range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 15 values: 19, 63, 58, 66, 61, 48, 28, 78, 57, 75, 59, 73, 30, 53, 65
Find the median and the interquartile range.

Ordered: 19, 28, 30, 48, 53, 57, 58, 59, 61, 63, 65, 66, 73, 75, 78
Median = 59, lower quartile = 48, upper quartile = 66
IQR = 66 − 48
Answer: median = 59, IQR = 18

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(4x^4)^3 \div 8x^{-2}$.

$(4x^4)^3 = 64x^{12}$
$64x^{12} \div 8x^{-2}$: divide the numbers and subtract the powers
$= 8x^{14}$
Answer: $8x^{14}$

4 Change of sign to locate a root (All boards)

$f(x) = e^x - 3x - 6$.
Show that $f(x) = 0$ has a root between $x = 2$ and $x = 3$.

$f(2) = -4.61$
$f(3) = 5.09$
f is continuous and there is a change of sign, so a root lies in the interval
Answer: f(2) = −4.61, f(3) = 5.09: change of sign

5 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^3 - x^2 + 4x + 12$ by $(x + 1)$.
Find the quotient and the remainder.

Use synthetic division with $a = -1$:
Bottom row: 1, −2, 6, 6
Quotient: $x^2 - 2x + 6$
Remainder: 6
Answer: Quotient $x^2 - 2x + 6$, remainder 6

6 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.
Use the small angle approximations to find an approximation for $\sin 5\theta \tan \theta$.

$\sin 5\theta \approx 5\theta$ and $\tan \theta \approx \theta$
So $\sin 5\theta \tan \theta \approx 5\theta^2$
Answer: $5\theta^2$

7 Friction: constant speed, find the coefficient (All boards)

A block of mass 10 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 19.6 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the block and ground.

R = 10 × 9.8 = 98
Constant speed: F = P = 19.6
F = μR: 19.6 = μ × 98
Answer: 0.2

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(25, p). H_0: p = 0.6, H_1: p > 0.6.
Find the critical region for a test at the 5% significance level.

P(X ≥ 20) = 0.0294 ≤ 0.05
P(X ≥ 19) = 0.0736 > 0.05
Answer: X ≥ 20

9 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car starts from rest and accelerates uniformly at 4 m s^{-2} for 10 s, then travels at constant speed for a further 20 s. Find the total distance travelled.

Speed reached: v = 4 × 10 = 40 m s^{-1}
Stage 1: $s = \frac{1}{2} \times 40 \times 10 = 200$
Stage 2: s = 40 × 20 = 800
Answer: 1000 m

10 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{2x - 3}{(x + 1)^2}$ in the form $\frac{A}{(x + 1)} + \frac{B}{(x + 1)^2}$.

2x − 3 = A(x + 1) + B
Coefficient of x: A = 2
Constant: −3 = −2 + B, so B = −5
Answer: A = 2, B = −5