Day 348 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 3 cos 4x
(b) e^{3x} + sin 4x

(a) dy/dx = −12 sin 4x
(b) dy/dx = 3 e^{3x} + 4 cos 4x
Answer: (a) −12 sin 4x (b) 3 e^{3x} + 4 cos 4x

2 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of the squares of two integers that differ by 5 is always odd.

Let the integers be n and n + 5.
$n^2 + (n + 5)^2 = 2n^2 + 10n + 25$
$= 2(n^2 + 5n) + 25$
$2(n^2 + 5n)$ is even and 25 is odd, so the sum is odd.
Answer: Shown

3 Particle held by a string and a horizontal force (All boards)

A particle of weight 60 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan\theta = \frac{5}{12}$. Find P and the tension T N.

Vertically: T cos θ = 60
Horizontally: T sin θ = P
Dividing: P = 60 tan θ; $T = \sqrt{60^2 + P^2}$
Answer: P = 25, T = 65

4 Double angle equation (All boards)

Solve $\sin 3x = -\frac{\sqrt{2}}{2}$ for 0° ≤ x ≤ 120°.

Let u = 3x, so 0° ≤ u ≤ 360°
$\sin u = -\frac{\sqrt{2}}{2}$: u = 225°, 315°, 585°, 675°, 945°, 1035°
Divide by 3: x = 75°, 105°
Answer: x = 75°, 105°

5 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 40 to 60 has frequency 15 and is drawn with a bar of height 2 cm.
The class 60 to 65 has frequency 15. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 15 ÷ 20 = 0.75, so 2 cm represents 0.75
Second bar: 15 ÷ 5 = 3
Height = $2 \times \frac{3}{0.75}$
Answer: 8 cm

6 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity (2**i** + 2**j**) m s^{-1} and constant acceleration (−2**i** + 2**j**) m s^{-2}.
Find its velocity after 3 s.

v = u + at
v = (2**i** + 2**j**) + 3(−2**i** + 2**j**)
= (2**i** + 2**j**) + (−6**i** + 6**j**)
Answer: (−4i** + 8**j**) m s^{-1}**

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7 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of data on height and arm span have r = 0.65.
Test, at the 5% significance level, for evidence of correlation. The critical value is ±0.7067.

H_0: ρ = 0, H_1: ρ ≠ 0
r = 0.65 is not beyond the critical value ±0.7067
Do not reject H_0
Answer: Do not reject H_0: there is not enough evidence of correlation

8 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{5x + 6}{(x + 3)^2}$ in the form $\frac{A}{(x + 3)} + \frac{B}{(x + 3)^2}$.

5x + 6 = A(x + 3) + B
Coefficient of x: A = 5
Constant: 6 = −15 + B, so B = −9
Answer: A = 5, B = −9

9 Stationary points of a cubic (All boards)

$y = x^3 - 6x^2 - 1$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 12x = 0$
x = 0: y = −1; x = 4: y = −33
$d^2y/dx^2 = 6x - 12$: at x = 0 it is −12 (maximum), at x = 4 it is 12 (minimum)
Answer: (0, −1) maximum; (4, −33) minimum

10 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are x = 1 and y = 2, so a = 1 and c = 2.

Through (0, 5): $5 = \frac{k}{0 - 1} + 2$

$\frac{k}{-1} = 3$, so k = −3
Answer: a = 1, c = 2, k = −3