Day 347 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is $(3, -4)$, so $y = a(x - 3)^2 - 4$.

The curve passes through $(0, 14)$:

$14 = a(-3)^2 - 4$, so $18 = 9a$

$a = 2$

Answer: $y = 2(x - 3)^2 - 4$

2 Solve basic trig equation (exact, degrees) (All boards)

Solve $\cos x = \frac{\sqrt{3}}{2}$ for $0° \leq x \leq 180°$.

Reference angle: $\cos 30° = \frac{\sqrt{3}}{2}$

cos is positive in the first and fourth quadrants.

Using the cos graph/CAST and the interval $0° \leq x \leq 180°$:

$x = 30°$

Answer: $x = 30°$

3 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-x^3 - x^2 - 2x + 3$ is divided by $(2x - 1)$.

Put $2x - 1 = 0$: $x = \frac{1}{2}$

Remainder $= f(\frac{1}{2}) = (-\frac{1}{8}) + (-\frac{1}{4}) + (-1) + 3$

$= \frac{13}{8}$

Answer: $\frac{13}{8}$

4 Moments: balancing a plank (AQA, Edexcel, OCR, MEI)

A light plank rests on a pivot. A child of mass 45 kg sits 3 m from the pivot on one side. Where must a child of mass 15 kg sit on the other side for the plank to balance horizontally?

Moments about the pivot:

$45\text{ g} \times 3 = 15\text{ g} \times d$

$d = \frac{45 \times 3}{15}$

Answer: 9 m from the pivot

5 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

For data set A, $r = -0.62$. For data set B, $r = -0.72$.

Which data set has the stronger linear correlation? Give a reason.

Strength depends on the size of r, ignoring the sign.

$|-0.62| = 0.62$ and $|-0.72| = 0.72$.

Answer: data set B, as $|r|$ is larger

6 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 5 and common difference 6. Find the smallest n for which the sum of the first n terms exceeds 1000.

$S_n = \frac{n}{2}[10 + 6(n - 1)] = \frac{n(6n + 4)}{2}$

$S_{17} = 901$ and $S_{18} = 1008$

The sum first exceeds 1000 when $n = 18$.

Answer: $n = 18$

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7 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(8\mathbf{i} + 4\mathbf{j})$ N and $(-6\mathbf{i})$ N act on a particle of mass 2 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(8\mathbf{i} + 4\mathbf{j}) + (-6\mathbf{i})$
= $(2\mathbf{i} + 4\mathbf{j})$ N

F = ma: $a = \frac{1}{2}(2\mathbf{i} + 4\mathbf{j})$

Answer: $(\mathbf{i} + 2\mathbf{j})$ m s^{-2}

8 Integrate using partial fractions (All boards)

$\frac{-5x - 7}{(x + 2)(x + 1)} \equiv \frac{A}{x + 2} + \frac{B}{x + 1}$.

Find A and B, and hence find $\int \frac{-5x - 7}{(x + 2)(x + 1)}\,dx$.

$-5x - 7 \equiv A(x + 1) + B(x + 2)$
$x = -2$ gives $A = -3$; $x = -1$ gives $B = -2$
$\int \left(\frac{-3}{x + 2} + \frac{-2}{x + 1}\right) dx = -3 \ln|x + 2| + -2 \ln|x + 1| + c$
Answer: A = −3, B = −2; −3 ln |x + 2| − 2 ln |x + 1| + c

9 Normal approximation to the binomial (CIE)

X ~ B(50, 0.3). Use a normal approximation to find P(X ≥ 15), to 3 s.f.

np = 15 and npq = 10.5, so use Y ~ N(15, 10.5)
Continuity correction: P(Y > 14.5)
$z = (14.5 - 15) \div \sqrt{10.5}$
Answer: 0.561

10 Trapezium rule from a function (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_0^3 \frac{1}{1 + x^2}\,dx$,

giving your answer to 3 significant figures.

$h = \frac{3 - 0}{4} = 0.75$; x = 0, 0.75, 1.5, 2.25, 3
y = 1.000, 0.6400, 0.3077, 0.1649, 0.1000
Area $\approx \frac{h}{2}$[first + last + 2(sum of the rest)] = $\frac{0.75}{2}$[1.000 + 0.1000 + 2 × 1.113]
= 1.25
Answer: 1.25