Day 345 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 2 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}.
Find (a) its acceleration, (b) its speed after 4 s.

(a) Parallel to slope: $mg\sin\alpha = ma$

$a = 9.8 \times \frac{3}{5}$

(b) $v = u + at = 0 + 5.88 \times 4$

Answer: (a) 5.88 m s^{-2} (b) 23.52 m s^{-1}

2 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 13 terms of the series
$-5 - 3 - 1 + 1 + \ldots$

$a = -5$, $d = 2$, $n = 13$

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{13}{2}[2(-5) + 12 \times 2]$

$= 91$

Answer: 91

3 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(30, p)$ and $E(X) = 15$.
Find p and $\mathrm{Var}(X)$.

$np = 15$, so $p = 15 \div 30 = 0.5$

$\mathrm{Var}(X) = npq = 15 \times 0.5$

Answer: $p = 0.5$, $\mathrm{Var}(X) = 7.5$

4 Partial fractions: find the constants (All boards)

Express $\frac{5x + 21}{(x+5)(x+1)}$ in the form $\frac{A}{x+5} + \frac{B}{x+1}$.
Find the values of A and B.

$5x + 21 \equiv A(x+1) + B(x+5)$

$x = -5$: $-4 = A(-4)$, so $A = 1$

$x = -1$: $16 = B(4)$, so $B = 4$

Answer: $A = 1$, $B = 4$

5 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^3 + 5x^2 + 4x - 14$ by $(x + 3)$.
Find the quotient and the remainder.

Use synthetic division with $a = -3$:
Bottom row: 1, 2, −2, −8

Quotient: $x^2 + 2x - 2$

Remainder: −8

Answer: Quotient $x^2 + 2x - 2$, remainder −8

6 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = -3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (-3)(1) + (-2)(4) + (-3)(-3) = -2$

$|\mathbf{a}| = \sqrt{22}$, $|\mathbf{b}| = \sqrt{26}$

$\cos\theta = \frac{-2}{\sqrt{22} \times \sqrt{26}}$, so $\theta = 94.8°$

Answer: 94.8°

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7 Car towing a caravan (All boards)

A car of mass 1500 kg tows a caravan of mass 600 kg on a level road with acceleration 0.75 m s^{-2}. Resistances: 200 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 200 - 100 = 2100 \times 0.75$

Caravan: $T - 100 = 600 \times 0.75$

Answer: D = 1875 N, T = 550 N

8 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = 1t$, $y = 6t^2 + 1$.

Find the gradient at the point where $t = 2$.

$dx/dt = 1$, $dy/dt = 12t$

$dy/dx = \frac{12t}{1}$

At $t = 2$: $dy/dx = 24$

Answer: 24

9 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.4$, H_1: $p \neq 0.4$. In a sample, $x = 14$.

Test at the 5% significance level.

Two-tailed test, so compare with $0.05 \div 2 = 0.025$

$P(X \geq 14) = 0.00647$

$0.00647 < 0.025$, so reject H_0

Answer: Reject H_0 ($P(X \geq 14) = 0.00647$)

10 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x - 3)$.

$y = f(x - 3)$ is obtained from $y = f(x)$ by a:

translation by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Answer: translation by the vector $\begin{pmatrix} \mathbf{3} \\ \mathbf{0} \end{pmatrix}$