Day 344 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (3)(2) + (-3)(-1) + (1)(2) = 11$

$|\mathbf{a}| = \sqrt{19}$, $|\mathbf{b}| = \sqrt{9}$

$\cos\theta = \frac{11}{\sqrt{19} \times \sqrt{9}}$, so $\theta = 32.7°$

Answer: 32.7°

2 Regression: interpreting coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

A model relates the mass, m kg, of a puppy to its age, w weeks by $m = 4 + 1.2w$.
Interpret the value 1.2 in this context.

The gradient is the change in m for each 1 unit of w.
For each extra week, m increases by 1.2 kg.
Answer: each extra week, m increases by 1.2 kg

3 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and moves in a straight line with constant acceleration 2 m s^{-2}.
It travels 49 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$

$49 = \frac{1}{2} \times 2 \times t^2$

$t^2 = 49$

Answer: 7 s

4 Poisson distribution (CIE)

Emails arrive at an average rate of 3.5 per hour, at random.
Find the probability of exactly 7 in 2 hours, to 3 s.f.

Mean for 2 hours = $2 \times 3.5 = 7$

$P(X = 7) = e^{-7} \times 7^7 \div 5040!$

Answer: 0.149

5 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations
$x = 2t - 3$, $y = 3t^2 - 5$.
The point P has y-coordinate -2 and positive t.
Find the coordinates of P.

$3t^2 - 5 = -2$, so $t^2 = 1$, giving $t = 1$ $(t > 0)$

$x = 2 \times 1 - 3 = -1$

Answer: (−1, −2)

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 9 m s^{-1}.
At that instant car B starts from rest at O with constant acceleration 2 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $9t = \frac{1}{2} \times 2 \times t^2$

$t(9 - 1t) = 0$, so $t = 0$ or $t = 9 \div 1$

Answer: 9 s

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7 Solve a^x = b using logarithms (All boards)

Solve $2^{x+2} = 200$.

Give your answer to 3 significant figures.

Take logs: $(x + 2) \log 2 = \log 200$

$x + 2 = 7.644$, so $x = 5.64$

Answer: $x = 5.64$

8 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |4x - 9|$. Write down the coordinates of the vertex and of the point where the graph meets the y-axis.

The vertex is on the x-axis where $4x - 9 = 0$: $x = \frac{9}{4}$

$x = 0$: $y = |-9| = 9$

Answer: Vertex $(\frac{9}{4}, 0)$; y-axis $(0, 9)$

9 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\cos\theta = \frac{15}{17}$.

Find the exact values of $\sin\theta$ and $\tan\theta$.

$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{225}{289} = \frac{64}{289}$

θ is acute, so $\sin\theta = \frac{8}{17}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{8}{15}$

Answer: $\sin\theta = \frac{8}{17}$, $\tan\theta = \frac{8}{15}$

10 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{-4}{x+2} - 1$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: $x = -2$ and $y = -1$.

$y = 0$: $\frac{-4}{x+2} = 1$, so $x = -6$

$x = 0$: $y = \frac{-4}{2} - 1 = -3$

Answer: $x = -2$, $y = -1$; $(-6, 0)$, $(0, -3)$