Day 343 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(4x^2)^2 \div 2x^{-1}$.

$(4x^2)^2 = 16x^4$

$16x^4 \div 2x^{-1}$: divide the numbers and subtract the powers

$= 8x^5$

Answer: $8x^5$

2 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-5\mathbf{i} - 12\mathbf{j})$ m s^{-1}.
At time t = 0 its position vector is $(5\mathbf{i} + 8\mathbf{j})$ m.
(a) Find its position vector when t = 3.
(b) Find the distance it travels in 3 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (5\mathbf{i} + 8\mathbf{j}) + 3(-5\mathbf{i} - 12\mathbf{j})$

(b) speed $= \sqrt{5^2 + 12^2} = 13$ m s^{-1}
distance $= 13 \times 3$
Answer: (a) $(-10\mathbf{i} - 28\mathbf{j})$ m (b) 39 m

3 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = 5\mathbf{i} - 12\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{5^2 + 12^2} = 13$

(b) Acute angle $= \tan^{-1}(\frac{12}{5}) = 67.4°$

p is in the fourth quadrant, so the angle is 292.6°
Answer: (a) 13 (b) 292.6°

4 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(150, 8^2)$. Given that $P(X < a) = 0.7$, find a to 3 s.f.

$P(Z < z) = 0.7$, so $z = 0.5244$ (from the inverse normal function)
$a = 150 + 0.5244 \times 8$
Answer: 154

5 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = 3x(4 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where x = 0 and x = 4

Area $= \int_0^4 3(4x - x^2)\,dx = [3(\frac{4x^2}{2} - \frac{x^3}{3})]$ from 0 to 4

$= 3(\frac{4^3}{2} - \frac{4^3}{3})$

$= 32$
Answer: 32

6 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form $px + qy = r$ with integers p, q and r, (b) the length AB.

Gradient $= \frac{-3 - 0}{0 - 3} = 1$

Using $y = mx + c$ with $c = -3$: $y = x - 3$, so $x - y = 3$

$AB^2 = 3^2 + 3^2 = 18$
Answer: (a) $x - y = 3$ (b) $3\sqrt{2}$

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7 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(-4\mathbf{i} - 2\mathbf{j})$ N and $(19\mathbf{i} + 7\mathbf{j})$ N act on a particle of mass 5 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(-4\mathbf{i} - 2\mathbf{j}) + (19\mathbf{i} + 7\mathbf{j})$

$= (15\mathbf{i} + 5\mathbf{j})$ N

F = ma: $a = \frac{1}{5}(15\mathbf{i} + 5\mathbf{j})$

Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}

8 Integrate using partial fractions (All boards)

$\frac{-3x - 4}{(x + 1)(x + 2)} \equiv \frac{A}{x + 1} + \frac{B}{x + 2}$.

Find A and B, and hence find $\int \frac{-3x - 4}{(x + 1)(x + 2)}\,dx$.

$-3x - 4 \equiv A(x + 2) + B(x + 1)$

$x = -1$ gives $A = -1$; $x = -2$ gives $B = -2$

$\int \left(\frac{-1}{x + 1} + \frac{-2}{x + 2}\right) dx = -1 \ln|x + 1| + -2 \ln|x + 2| + c$

Answer: $A = -1$, $B = -2$; $-\ln|x + 1| - 2 \ln|x + 2| + c$

9 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = kx^3$ for $0 \leq x \leq 3$, and 0 otherwise. Find the value of k.

$\int$ from 0 to 3 of $k\,x^3\,dx = 1$

$k \times \frac{3^4}{4} = 1$

Answer: $k = \frac{4}{81}$

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 3x + k$ is a tangent to the curve $y = x^2 + 7x + 4$.

Find the value of k.

$x^2 + 7x + 4 = 3x + k$

$x^2 + 4x + (4 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(4 - k) = 0$

Answer: $k = 0$