Day 342 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 F = ma: find the acceleration (All boards)

A horizontal force of 8 N acts on a sledge of mass 20 kg on a smooth horizontal surface.
Find the acceleration.

F = ma
8 = 20 a
Answer: 0.4 m s^{-2}

2 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of y = f(x + 4).

y = f(x + 4) is obtained from y = f(x) by a:

translation by the vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

Answer: translation by the vector $\begin{pmatrix} \mathbf{-4} \\ \mathbf{0} \end{pmatrix}$

3 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 3 : 1.
Find AB and AP in terms of $\mathbf{a}$ and $\mathbf{b}$.

AB = OB − OA = $\mathbf{b} - \mathbf{a}$

AP = $\frac{3}{4}$ AB = $\frac{3}{4}(\mathbf{b} - \mathbf{a})$

Answer: AB = $\mathbf{b} - \mathbf{a}$; AP = $\frac{3}{4}(\mathbf{b} - \mathbf{a})$

4 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Rationalise the denominator of $\frac{2}{5 - \sqrt{7}}$.

Write your answer in the form $p + q\sqrt{7}$.

Multiply top and bottom by $5 + \sqrt{7}$:

$$\frac{2(5 + \sqrt{7})}{(5)^2 - (\sqrt{7})^2} = \frac{2(5 + \sqrt{7})}{18}$$

Answer: $\frac{5}{9} + \frac{1}{9}\sqrt{7}$

5 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(p\mathbf{i} + q\mathbf{j})$ N, $(-3\mathbf{i} - \mathbf{j})$ N and $(-\mathbf{i} - 2\mathbf{j})$ N act on a particle which is in equilibrium. $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:
i components: p + (−3) + (−1) = 0
j components: q + (−1) + (−2) = 0
Answer: p = 4, q = 3

6 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The nth term of an arithmetic sequence is 3n + 4.
Find the smallest n for which the term exceeds 500.

3n + 4 > 500

n > $\frac{496}{3}$ = 165.33...

The smallest integer n is 166.
Answer: n = 166

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7 Rewrite a sin x + b cos x in R form (All boards)

Express 7 sin x + 24 cos x in the form R cos(x − α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

R cos(x − α) = R cos x cos α + R sin x sin α
R cos α = 24 and R sin α = 7
R = $\sqrt{7^2 + 24^2}$ = 25, tan α = $\frac{7}{24}$, α = 16.3°

Answer: 25 cos(x − 16.3°)

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(25, p). H_0: p = 0.25, H_1: p < 0.25.
Find the critical region for a test at the 1% significance level.

P(X ≤ 1) = 0.00702 ≤ 0.01
P(X ≤ 2) = 0.0321 > 0.01
Answer: X ≤ 1

9 Definite integrals (exact) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} 6 \sin 2x \, dx$.

[−3 cos 2x] from 0 to $\frac{\pi}{2}$
= −3 cos π − (−3 cos 0) = 3 + 3
Answer: 6

10 Normal approximation to the binomial (CIE)

X ~ B(50, 0.25). Use a normal approximation to find P(X ≥ 12), to 3 s.f.

np = 12.5 and npq = 9.375, so use Y ~ N(12.5, 9.375)
Continuity correction: P(Y > 11.5)
z = (11.5 − 12.5) ÷ $\sqrt{9.375}$
Answer: 0.628