Day 341 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a boat is $(-4\mathbf{i} + 3\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{4^2 + 3^2} = \sqrt{25}$

(b) $\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(\frac{3}{4})$

Answer: (a) 5 m s^{-1} (b) 36.9°

2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(4x^4)^3 \div 16x$.

$(4x^4)^3 = 64x^{12}$

$64x^{12} \div 16x$: divide the numbers and subtract the powers

$= 4x^{11}$

Answer: $4x^{11}$

3 Differentiating velocity to find acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v m s^{-1}, where $v = 3t^2 - 18t + 2$.
(a) Find its acceleration when t = 3.
(b) Find the time when its acceleration is zero.

$a = \frac{dv}{dt} = 6t - 18$

(a) $t = 3$: $a = 6 \times 3 - 18$

(b) $6t - 18 = 0$, so $t = \frac{18}{6}$

Answer: (a) 0 m s^{-2} (b) 3 s

4 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that exactly one of R and L happens.

Exactly one: (R and L′) or (R′ and L)

$= 0.5 \times 0.6 + 0.5 \times 0.9$

Answer: 0.75

5 Stationary points of a cubic (All boards)

$y = x^3 - 3x^2$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 6x = 0$

$x = 0$: $y = 0$; $x = 2$: $y = -4$

$d^2y/dx^2 = 6x - 6$: at $x = 0$ it is -6 (maximum), at $x = 2$ it is 6 (minimum)

Answer: (0, 0) maximum; (2, −4) minimum

6 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t + 3$, $y = 3t^2 - 4$.

Find a Cartesian equation of the curve.

From $x = t + 3$: $t = x - 3$

Substitute into y: $y = 3(x - 3)^2 - 4$

Answer: $y = 3(x - 3)^2 - 4$

Day 341 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 5) and (4, $\frac{5}{16}$).

Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 5$

$x = 4$: $5 \times b^4 = \frac{5}{16}$, so $b^4 = \frac{1}{16}$

$b = \frac{1}{2}$

Answer: $a = 5$, $b = \frac{1}{2}$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.2. If so, the probability that the patient recovers is 0.9; otherwise it is 0.1. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.2 × 0.9 = 0.18; P(not first and second) = 0.8 × 0.1 = 0.08

P(second) = 0.18 + 0.08 = 0.26

P = 0.18 ÷ 0.26

Answer: $\frac{9}{13}$ = 0.692

9 Integration by parts (All boards)

Use integration by parts to find

$\int x \sin x \, dx$.

$u = x$, $dv/dx = \sin x$; $du/dx = 1$, $v = -\cos x$

$\int u \, (dv/dx) \, dx = uv - \int v \, (du/dx) \, dx$

Evaluate the remaining integral and simplify

Answer: $-x \cos x + \sin x + c$

10 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + x^2 - 22x - 40$

(a) Show that (x + 2) is a factor of f(x).

(b) Factorise f(x) completely.

$f(-2) = (-8) + 4 + 44 + (-40) = 0$, so (x + 2) is a factor.

$f(x) = (x + 2)(x^2 - x - 20)$

$x^2 - x - 20 = (x + 4)(x - 5)$

Answer: (b) $f(x) = (x + 4)(x + 2)(x - 5)$