Day 340 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Integrate standard functions (All boards)

Find

(a) $\int 8 \sec^2 2x \, dx$

(b) $\int 8 e^{2x} \, dx$

Use the standard integrals and divide by the coefficient of x where needed.

(a) = 4 tan 2x + c

(b) = 4 e^{2x} + c

Answer: (a) 4 tan 2x + c (b) 4 e^{2x} + c

2 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

M(−1, −5) is the midpoint of the line segment AB.

A is the point (−4, −5). Find the coordinates of B.

x-coordinate: $\frac{-4 + x_B}{2} = -1$, so $x_B = 2$

y-coordinate: $\frac{-5 + y_B}{2} = -5$, so $y_B = -5$

Answer: B(2, −5)

3 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 20 values, $\Sigma x = 560$ and $\Sigma x^2 = 16060$.

Find the mean and the standard deviation.

Mean = 560 ÷ 20 = 28

Variance = $\frac{16060}{20} - 28^2 = 803 - 784 = 19$

Standard deviation = $\sqrt{19}$

Answer: mean = 28, standard deviation = 4.36

4 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = \sqrt{x + 4}$ from x = 0 to x = 6 divided into 3 strips of equal width.

Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 2; y values: 2.000, 2.449, 2.828, 3.162

Area ≈ $\frac{2}{2}[2.000 + 3.162 + 2(5.278)]$

= 15.7

Answer: 15.7

5 Work done and kinetic energy (CIE)

A particle of mass 10 kg changes its speed from 4 m s^{-1} to 10 m s^{-1}. Find the increase in its kinetic energy.

KE = $\frac{1}{2}mv^2$

Increase = $\frac{1}{2} \times 10 \times 10^2 - \frac{1}{2} \times 10 \times 4^2$

Answer: 420 J

6 Integration by substitution (All boards)

Use the substitution $u = x^2 + 3$ to find

$\int \frac{4x}{x^2 + 3} \, dx$.

$u = x^2 + 3$, $du = 2x \, dx$, so $x \, dx = \frac{1}{2} \, du$

$\int \frac{4}{2u} \, du = \frac{4}{2} \ln |u|$

Answer: 2 ln (x^2 + 3) + c

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7 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-3x + 10}{(x + 2)(x - 2)}$ in partial fractions.

Write $\frac{-3x + 10}{(x + 2)(x - 2)} = \frac{A}{(x + 2)} + \frac{B}{(x - 2)}$

$-3x + 10 = A(x - 2) + B(x + 2)$

$x = -2$: $16 = A(-4)$ so $A = -4$

$x = 2$: $4 = B(4)$ so $B = 1$

Answer: $\mathbf{\frac{-4}{(x + 2)} + \frac{1}{(x - 2)}}$

8 Rough slope: limiting equilibrium or sliding (All boards)

A particle is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan \alpha = \frac{20}{21}$. Find the coefficient of friction.

Perpendicular: $R = mg \cos \alpha$

Parallel (limiting): $F = mg \sin \alpha$

$\mu = \frac{F}{R} = \tan \alpha$

Answer: $\mathbf{\frac{20}{21}}$

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x + k$ is a tangent to the curve $y = x^2 + 2x + 1$.

Find the value of k.

$x^2 + 2x + 1 = -2x + k$

$x^2 + 4x + (1 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(1 - k) = 0$

Answer: $k = -3$

10 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

The product moment correlation coefficient between hours of sunshine and ice cream sales is r = 0.62. Interpret this value.

r is close to 1, so there is moderate positive linear correlation.

As hours of sunshine increases, ice cream sales tends to increase.

Answer: moderate positive linear correlation